

Birzeit University
Mathematics Department
Math 132
Final Exam
First Summer Semester 2012/2013

Student Name:

Student Number:

Time: 150 minutes

There are 4 questions in 10 pages

Question 1. (60%) Circle the most correct answer:

(1) The volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$, $x = 1$, and the x -axis, about the y -axis, is:

(a) $\frac{3\pi}{5}$

(b) $\frac{\pi}{5}$

(c) $\frac{2\pi}{5}$

(d) $\frac{4\pi}{5}$

(2) $\sum_{n=2}^{\infty} (0.5)^{-n} =$

(a) 2

(b) 1

(c) $\frac{1}{2}$

(d) None of the above

(3) $\int_0^{\frac{\pi}{2}} \tan x \, dx =$

(a) 0

(b) -1

(c) ∞

(d) $-\infty$

(4) If y is the solution of the differential equation $\frac{dy}{dx} = 3x^2y + y$, $y(1) = e$, then $y(-1) =$

(a) -1

(b) -3

(c) e^{-1}

(d) e^{-3}

(5) $\int_1^4 \frac{3\sqrt{x}}{2\sqrt{x}} dx =$

(a) $\frac{6}{\ln 3}$

(b) $\frac{3}{\ln 3}$

(c) $\frac{78}{\ln 3}$

(d) $\frac{9}{\ln 3}$

(6) The volume of the solid whose base is the region enclosed between the curves $y = x^2$ and $y = x$, and whose cross sections perpendicular to the x -axis are equilateral triangles of height 4, is:

(a) $\frac{1}{2}$

(b) $\frac{1}{3}$

(c) $\frac{1}{6}$

(d) $\frac{1}{4}$

(7) If $a_n = n3^{\frac{1}{n}}$, $n \in \mathbb{N}$, then $\lim_{n \rightarrow \infty} a_n =$

(a) 1

(b) 0

(c) ∞

(d) $\ln 3$

(8) $\sum_{n=2}^{\infty} \frac{2n-1}{n^2(n-1)^2} =$

(a) -1

(b) 1

(c) $\frac{1}{4}$

(d) 2

(9) Assuming its convergence, find the limit of the following recursively defined sequence, $a_1 = 8$,
 $a_{n+1} = \sqrt{a_n + 8} - 2$:

(a) 1

(b) -4

(c) -2

(d) 8

(10) $\int e^{\sqrt{2x+1}} dx =$

(a) $2\sqrt{2x+1} e^{\sqrt{2x+1}} + C$

(b) $\frac{e^{\sqrt{2x+1}}}{2\sqrt{2x+1}} + C$

(c) $\sqrt{2x+1} e^{\sqrt{2x+1}} - e^{\sqrt{2x+1}} + C$

(d) $\sqrt{2x+1} e^{\sqrt{2x+1}} - \sqrt{2x+1} + C$

(11) If $\tanh x = \frac{1}{2}$, $x < 0$, then $\operatorname{sech} x =$

(a) $\frac{\sqrt{5}}{2}$

(b) $\frac{-\sqrt{5}}{2}$

(c) $\frac{\sqrt{3}}{2}$

(d) $\frac{-\sqrt{3}}{2}$

(12) Which one of the following functions is the fastest growing as $x \rightarrow \infty$:

(a) $e^{\frac{x}{3}}$

(b) $\ln(\ln x)$

(c) 3^x

(d) $4 + 2^x$

(13) The series $\sum_{n=0}^{\infty} \frac{3^n}{5^n + 2^n}$:

(a) Converges by direct comparison with $\sum_{n=0}^{\infty} \frac{3^n}{4^n}$

(b) Converges by direct comparison with $\sum_{n=0}^{\infty} \frac{1}{2^n}$

(c) Converges by direct comparison with $\sum_{n=0}^{\infty} \frac{3^n}{7^n}$

(d) Converges by summing its terms as a geometric series

(14) The series $\sum_{n=2}^{\infty} \frac{(n+1) \ln n}{\sqrt{n}}$:

(a) Converges by the integral test

(b) Converges by direct comparison with $\sum_{n=2}^{\infty} \frac{1}{n^2}$

(c) Diverges by the ratio test

(d) Diverges by the n th-term test

(15) If $a_n = \left(1 - \frac{2}{n}\right)^{\frac{n}{2}}$, $n \in \mathbb{N}$, then $\lim_{n \rightarrow \infty} a_n =$

- (a) e^{-2}
- (b) e^{-1}
- (c) e^{-4}
- (d) $e^{-\frac{1}{2}}$

(16) The series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n(n+2)(n+3)}}$

- (a) Converges by direct comparison with $\sum_{n=1}^{\infty} \frac{1}{\sqrt[5]{n^4}}$
- (b) Converges by limit comparison with $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3}}$
- (c) Converges by direct comparison with $\sum_{n=1}^{\infty} \frac{1}{\sqrt[4]{n^3}}$
- (d) Diverges by the ratio test

(17) $i^{215} =$

- (a) i
- (b) $-i$
- (c) 1
- (d) -1

(18) The integral $\int_2^{\infty} \frac{dx}{\sqrt[3]{x}(\sqrt{x}-1)}$:

- (a) Converges by limit comparison with $\int_2^{\infty} \frac{dx}{\sqrt[3]{x}}$
- (b) Converges by limit comparison with $\int_2^{\infty} \frac{dx}{\sqrt{x}}$
- (c) Diverges by direct comparison with $\int_2^{\infty} \frac{dx}{\sqrt[3]{x^2}}$
- (d) Diverges by direct comparison with $\int_2^{\infty} \frac{dx}{\sqrt[5]{x^5}}$

(19) The radius of convergence of the series $\sum_{n=1}^{\infty} \frac{e^n (x-1)^n}{n^2 3^n}$ is:

- (a) $\frac{3}{e} + 1$
- (b) $\frac{e}{3} + 1$
- (c) $\frac{3}{e}$
- (d) $\frac{e}{3}$

(20) $\int_0^1 x^2 \ln x \, dx =$

- (a) $-\frac{1}{4}$
- (b) $-\frac{1}{9}$
- (c) ∞
- (d) $-\infty$

(21) The series $\sum_{n=1}^{\infty} \frac{(-1)^n (\sqrt{n} + n)}{\sqrt{n^5 + 1}}$:

- (a) Converges absolutely
- (b) Converges conditionally

(c) Diverges by limit comparison with $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3}}$

(d) Diverges by the n th-term test

(22) $\int_1^{\sqrt{3}} \frac{dx}{x\sqrt{x^2+1}} =$

- (a) $\ln \left(\frac{\sqrt{3}+1}{\sqrt{2}} \right)$
- (b) $\ln \left(\frac{\sqrt{2}}{\sqrt{3}+1} \right)$
- (c) $\ln \left(\frac{\sqrt{2}+1}{\sqrt{3}} \right)$
- (d) $\ln \left(\frac{\sqrt{3}}{\sqrt{2}+1} \right)$

(23) $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sec^2 x \tan^2 x dx =$

(a) $\frac{-26}{9\sqrt{3}}$

(b) $\frac{28}{9\sqrt{3}}$

(c) $\frac{-13}{3\sqrt{3}}$

(d) $\frac{20}{3\sqrt{3}}$

(24) A partial fraction for the function $f(x) = \frac{3x+1}{x^3-8}$ is:

(a) $\frac{A}{x-2} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$

(b) $\frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{x+2}$

(c) $\frac{A}{x-2} + \frac{Bx+C}{x^2-2x+4}$

(d) $\frac{A}{x-2} + \frac{Bx+C}{x^2+2x+4}$

(25) The series $\sum_{n=2}^{\infty} \left(\frac{n}{n^2-1} \right)^{n^2}$:

(a) Converges by summing its terms as a telescoping series

(b) Converges by the n th-term test

(c) Converges by the root test

(d) Diverges by direct comparison with $\sum_{n=2}^{\infty} \frac{1}{\sqrt[n]{n}}$

(26) $\frac{4-i}{1+i} =$

(a) $\frac{3}{2} - \frac{5}{2}i$

(b) $\frac{3}{2} + \frac{5}{2}i$

(c) $\frac{5}{2} - \frac{3}{2}i$

(d) $\frac{5}{2} + \frac{3}{2}i$

(27) The series $\sum_{n=3}^{\infty} \frac{\sqrt{\ln n}}{n^{1.1}}$:

- (a) Converges by limit comparison with $\sum_{n=3}^{\infty} \frac{1}{n^{1.05}}$
(b) Converges by limit comparison with $\sum_{n=3}^{\infty} \frac{1}{n^{1.1}}$
(c) Converges by direct comparison with $\sum_{n=3}^{\infty} \frac{1}{n^{0.95}}$
(d) Diverges by the ratio test

(28) If $x = \ln(\sec t + \tan t)$, $y = t \sec t$, $-\frac{\pi}{2} < t < \frac{\pi}{2}$, then $\left. \frac{d^2 y}{dx^2} \right|_{t=\frac{\pi}{4}} =$

- (a) $\frac{\pi}{2} + 1$
(b) $\frac{\pi}{2\sqrt{2}} + \frac{1}{\sqrt{2}}$
(c) $\frac{\pi}{4} + 1$
(d) None of the above

(29) The series $\sum_{n=1}^{\infty} (-1)^n (\sqrt{n+1} - \sqrt{n-1})$:

- (a) Converges absolutely
(b) Converges conditionally
(c) Diverges by the n th-term test
(d) Diverges by direct comparison with $\sum_{n=1}^{\infty} (-1)^n \sqrt{n}$

(30) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(n+1)2^{n+1}} =$

- (a) $\ln\left(\frac{2}{3}\right)$
(b) $\ln\left(\frac{1}{3}\right)$
(c) $\ln\left(\frac{3}{2}\right)$
(d) $\ln\left(\frac{3}{4}\right)$

Question 2. (15%) (a) Use the binomial series to find out the first four nonzero terms of the Maclaurin series of $(1+x)^{\frac{2}{3}}$, $-1 < x < 1$.

(b) (1) Find the Taylor series of $f(x) = \tan^{-1}(3x^2)$, about $a = 0$, and specify its interval of convergence.

2017



2016

(2) Use the above series to estimate the value of $\tan^{-1}\left(\frac{1}{3}\right)$ with an error of magnitude less than 0.001.

Question 3. (13%) (a) Find the length of the parametric curve:

$$x = t, \quad y = \frac{t^2}{2}, \quad 0 \leq t \leq 1.$$

(b) Sketch the parametric curve defined by the equations:

$$x = 3 \cos t, \quad y = 2 \sin t, \quad -\frac{\pi}{2} \leq t \leq \pi.$$

Question 4. (12%) (a) Find the four fourth roots of -81 .

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(b) Solve the equation: $2|z - 1 - i| = |z + \bar{z} - 2|$.

مجلس الطلبة

PROBLEM 1: 10 MARKS

Find the four fourth roots of $-8+8\sqrt{3}i$

$$\text{The modulus} = \sqrt{(-8)^2 + (8\sqrt{3})^2} = 16$$

$$\cos \theta = \frac{-8}{16} = -\frac{1}{2} \Rightarrow \theta = \cos^{-1}\left(-\frac{1}{2}\right) = 120^\circ = \frac{2\pi}{3}$$

$$w_k = 16^{1/4} \left[\cos\left(\frac{120^\circ + 2k \cdot 180^\circ}{4}\right) + i \sin\left(\frac{120^\circ + 2k \cdot 180^\circ}{4}\right) \right] \quad k=0,1,2,3$$

$$w_0 = 2 \left[\cos(30^\circ) + i \sin(30^\circ) \right] = 2 \left[\frac{\sqrt{3}}{2} + \frac{1}{2}i \right] = \sqrt{3} + i$$

$$w_1 = 2 \left[\cos(120^\circ) + i \sin(120^\circ) \right] = 2 \left[-\frac{1}{2} + \frac{i\sqrt{3}}{2} \right] = -1 + i\sqrt{3}$$

$$w_2 = 2 \left[\cos(210^\circ) + i \sin(210^\circ) \right] = 2 \left[-\frac{\sqrt{3}}{2} - \frac{1}{2}i \right] = -\sqrt{3} - i$$

$$w_3 = 2 \left[\cos(300^\circ) + i \sin(300^\circ) \right] = 2 \left[\frac{1}{2} - \frac{i\sqrt{3}}{2} \right] = 1 - i\sqrt{3}$$

$$w_0 = \sqrt{3} + i$$

$$w_1 = -1 + i\sqrt{3}$$

$$w_2 = -\sqrt{3} - i$$

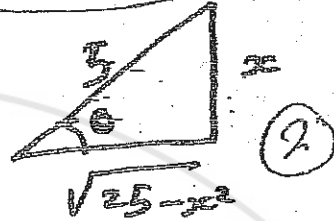
$$w_3 = 1 - i\sqrt{3}$$

PROBLEM 2: 10 MARKS EVALUATE

$$\int \frac{1}{x^2 \sqrt{25-x^2}} dx$$

$$\frac{x = 5 \sin \theta \quad (2)}{dx = 5 \cos \theta d\theta}$$

$$\int \frac{5 \cos \theta d\theta}{25 \sin^2 \theta \sqrt{25-25 \sin^2 \theta}} \quad (2)$$



$$\int \frac{5 \cos \theta d\theta}{25 \sin^2 \theta (5 \cos \theta)} \quad (2)$$

$$\frac{1}{25} \int \csc^2 \theta d\theta$$

$$= -\frac{1}{25} \cot(\theta) + C \quad (2)$$

$$= -\frac{1}{25} \frac{\sqrt{25-x^2}}{x} + C$$

535 M195051M535

PROBLEM 3(10 MARKS): CONSIDER s IS CONSTANT EVALUATE

$$\int_0^{\infty} t^2 e^{-st} dt$$

$$\begin{array}{l} t^2 \quad e^{-st} \\ \downarrow \quad \downarrow \\ 2t \quad -\frac{e^{-st}}{s} \\ \downarrow \quad \downarrow \\ 2 \quad -\frac{e^{-st}}{s^2} \\ \downarrow \quad \downarrow \\ 0 \quad -\frac{e^{-st}}{s^3} \end{array}$$

$$\lim_{\beta \rightarrow \infty} \int_0^{\beta} t^2 e^{-st} dt$$

$$= \lim_{\beta \rightarrow \infty} \left[-\frac{t^2}{s e^{st}} - \frac{2t}{s^2 e^{st}} - \frac{2}{s^3 e^{st}} \right]_0^{\beta}$$

$$= \lim_{\beta \rightarrow \infty} \left[\frac{-\beta^2}{s e^{s\beta}} - \frac{2\beta}{s^2 e^{s\beta}} - \frac{2}{s^3 e^{s\beta}} - (0 - 0 - \frac{2}{s^3}) \right]$$

$$= \frac{2}{s^3}$$

PROBLEM 4 (10 MARKS) DISCUSS THE CONVERGENCE

$$\sum_{k=0}^{\infty} \frac{k^2}{2^{3k}} (x+4)^k$$

By Ratio test

$$\lim_{k \rightarrow \infty} \left| \frac{\frac{(k+1)^2}{2^{3(k+1)}} (x+4)^{k+1}}{\frac{k^2}{2^{3k}} (x+4)^k} \right| < 1$$

$$= \lim_{k \rightarrow \infty} \frac{(k+1)^2}{k^2} \left(\frac{1}{2^3} \right) |x+4| < 1$$

$$= \lim_{k \rightarrow \infty} \frac{1}{8} |x+4| < 1$$

$$= |x+4| < 8 \longrightarrow \boxed{R=9}$$

$$-8 < x+4 < 8$$

$$\boxed{-12 < x < 4}$$

when $\boxed{x = -12}$ $\sum_{k=0}^{\infty} \frac{k^2}{2^{3k}} (-12+4)^k = \sum_{k=0}^{\infty} \frac{k^2}{(8)^k} (8)^k = \sum_{k=0}^{\infty} (-1)^k k^2$

$$\lim_{k \rightarrow \infty} k^2 = \infty \text{ div.}$$

when $\boxed{x = 4}$ $\sum_{k=0}^{\infty} \frac{k^2}{2^{3k}} (4+4)^k = \sum_{k=0}^{\infty} k^2$

$$\lim_{k \rightarrow \infty} k^2 = \infty \text{ div.}$$

The interval of conv. $(-12, 4)$

PROBLEM 5: 60 MARKS

Consider the test form the first column and the result form the second column

- | | |
|-------------------------------|----------------------------|
| 1. geometric series | a. converges absolutely |
| 2. p- series | b. converges conditionally |
| 3. telescoping series | c. diverges |
| 4. the nth-term test | |
| 5. the integral test | |
| 6. alternating series test | |
| 7. the direct comparison test | |
| 8. the limit comparison test | |
| 9. the ratio test | |
| 10. the root test | |

solve in details then circle the correct answer

1. radius of conv.

$$\sum_{k=1}^{\infty} \frac{2^k (x-3)^k}{\sqrt{k+3}} =$$

$$\lim \left| \frac{2^{k+1} (x-3)^{k+1} \sqrt{k+3}}{\sqrt{k+4} \cdot 2^k (x-3)^k} \right| < 1$$

$$= \lim 2|x-3| \frac{\sqrt{k+3}}{\sqrt{k+4}} < 1$$

$$= 2|x-3| < 1$$

$$|x-3| < \frac{1}{2}$$

$-\frac{1}{2} < x-3 < \frac{1}{2}$
 $\frac{5}{2} < x < \frac{7}{2}$
 $x = \frac{5}{2}$ by conv. by alternating
 $x = \frac{7}{2}$ div. by limit comp. test

a. $\frac{1}{2}$

b. $\frac{7}{2}$

c. $\frac{19}{6}$

d. $\frac{5}{2}$

2. $\sum_{k=1}^{\infty} \frac{\sin k}{k^2 + 1}$

$$a_k = \left| \frac{\sin k}{k^2 + 1} \right| \leq \frac{1}{k^2 + 1} \leq \frac{1}{k^2}$$

$\sum \frac{1}{k^2}$ conv. p. ser
 $\sum a_k$ conv. by D.C.T.

- a. divergent by p- series
 b. divergent by geometric series
 c. converges conditionally
 d. converges absolutely

3. $\sum_{k=0}^{\infty} \left(\frac{-1}{\sqrt{k+1}} \right)^k =$

- a. divergent by the ratio test
 b. divergent by the nth-root test
 c. converges conditionally
 d. converges absolutely

1) Let $a_k = \frac{1}{\sqrt{k+1}} > 0$

2) $f(x) = \frac{1}{\sqrt{x+1}} = (x+1)^{-1/2}$

$$f'(x) = \frac{-1}{2\sqrt{(x+1)^3}} < 0$$

decreasing series

3) $\lim a_k = 0$

$\sum (-1)^k a_k$ conv. alternating series

Let $b_k = \frac{1}{\sqrt{k}} = \frac{1}{k^{1/2}}$

$\frac{a_k}{b_k} = \lim \frac{\sqrt{k}}{\sqrt{k+1}} = 1$

b_k div. p-series

$\Rightarrow \sum a_k$ div. by limit C.T.

$$4. \sum_{k=2}^{\infty} \frac{(-1)^k \sqrt{k}}{\ln k} =$$

- a. divergent by the direct comparison test
- b. divergent by the kth-term test**
- c. converges absolutely
- d. converges conditionally

$$\lim_{k \rightarrow \infty} \frac{\sqrt{k}}{\ln k} = \lim_{k \rightarrow \infty} \frac{1}{\frac{2\sqrt{k}}{k}} = \lim_{k \rightarrow \infty} \frac{\sqrt{k}}{2} \neq 0$$

By k-th term test

the series div.

$$5. \sum_{k=1}^{\infty} \frac{\sqrt{n^2-1}}{n^3+2n^2+5} =$$

- a. convergent by the integral test
- b. divergent by limit comparison test
- c. convergent by limit comparison test**
- d. divergent by direct comparison test

Let $a_n = \frac{\sqrt{n^2-1}}{n^3+2n^2+5}$

$b_n = \frac{1}{n^2}$

$$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = \frac{\sqrt{n^2-1}}{n^3+2n^2+5} \cdot n^2 =$$

But $\sum b_n = \sum \frac{1}{n^2}$
Conv. p-series

$\therefore \sum a_n$ Conv. Limit C.T.

$$6. \sum_{k=0}^{\infty} \frac{(-2)^{3k-1}}{9^k} = \sum_{k=0}^{\infty} \frac{(-2)^{-1} (-2)^{3k}}{9^k} = \sum_{k=0}^{\infty} -\frac{1}{2} \left(\frac{-8}{9}\right)^k$$

a. $\frac{-2}{9}$

b. $\frac{9}{7}$

c. $\frac{-9}{7}$

d. $\frac{-9}{34}$

$$= -\frac{1}{2} \left[\frac{1}{1 - (-\frac{8}{9})} \right]$$

$$= -\frac{1}{2} \left[\frac{1}{1 + \frac{8}{9}} \right]$$

$$= -\frac{1}{2} \left[\frac{9}{9+8} \right] = \frac{-9}{34}$$

$$7. \sum_{k=1}^{\infty} \frac{e^k}{2^n} =$$

$$\int_1^{\infty} \frac{e^{1/x}}{x^2} dx$$

$u = \frac{1}{x} \mid x \rightarrow \infty \rightarrow u \rightarrow 0$
 $x \rightarrow 1 \rightarrow u \rightarrow 1$
 $du = -\frac{1}{x^2} dx$

- a. convergent by the integral test**
- b. divergent by limit comparison test
- c. convergent by telescoping series
- d. divergent by direct comparison test

$$= \int_0^1 e^u du = e^u \Big|_0^1$$

$$= \int_0^1 e^u du = e^u \Big|_0^1$$

$$= e - 1$$

$e^{1/x}$ cont., +ve, dec.

$$8. \sum_{k=1}^{\infty} \frac{\cos\left(\frac{k\pi}{6}\right)}{k\sqrt{k}} =$$

$$a_k \leq \frac{|\cos k\pi|}{k\sqrt{k}} \leq \frac{1}{k^{3/2}} = b_k$$

$\sum b_k$ conv. p-series $\Rightarrow \sum a_k$ conv by D.C.T.

- a. divergent by the direct comparison test
- b. divergent by the kth-term test
- c. converges absolutely
- d. converges conditionally

$$9. \lim_{x \rightarrow 0} (\cos 3x)^{\frac{1}{x}} =$$

$$\text{Let } y = (\cos 3x)^{\frac{1}{x}} \Rightarrow \ln y = \frac{\ln(\cos 3x)}{x}$$

$$\ln y = \frac{\ln(\cos 3x)}{x}$$

$$\lim_{x \rightarrow 0} \ln y = \lim_{x \rightarrow 0} \frac{\ln(\cos 3x)}{x} = \lim_{x \rightarrow 0} \frac{-\frac{1}{\cos 3x} \cdot (-3\sin 3x)}{1} = 0$$

$$\lim y = \lim e^{\ln y} = e^0 = 1$$

- a. 1
- b. 0
- c. $\frac{1}{2}$
- d. ∞

$$10. \lim_{x \rightarrow 0} (1-2x)^{\frac{1}{x}} =$$

$$\text{Let } y = (1-2x)^{\frac{1}{x}}$$

$$\ln y = \frac{\ln(1-2x)}{x}$$

$$\lim_{x \rightarrow 0} \frac{\ln(1-2x)}{x}$$

$$\lim_{x \rightarrow 0} \frac{-2}{1-2x} = -2$$

$$\Rightarrow \lim_{x \rightarrow 0} (1-2x)^{\frac{1}{x}} = \lim_{x \rightarrow 0} e^{-2} = e^{-2}$$

- a. does not exist
- b. e^{-2}
- c. e^{-2}
- d. $\frac{2}{e}$

$$11. \text{ the sequence } a_n = \frac{\ln(2+e^n)}{3n}$$

$$\lim_{n \rightarrow \infty} \frac{\ln(2+e^n)}{3n}$$

$$\lim_{n \rightarrow \infty} \frac{e^n}{2+e^n} = \lim_{n \rightarrow \infty} \frac{e^n}{6+3e^n}$$

$$= \lim_{n \rightarrow \infty} \frac{e^n}{3e^n} = \frac{1}{3}$$

- a. converges to $\frac{2}{3}$
- b. converges to $\frac{1}{3}$
- c. converges to 0
- d. divergent sequences

12. $\int \frac{2x^2 - 3x + 2}{x^3 + x}$ CAN BE INTEGRATED BY PARTIAL FRACTION

- a. $\frac{A}{x} + \frac{Bx + C}{x^2 + 1}$
 b. $\frac{A}{x} + \frac{B}{x^2 + 1}$
 c. $\frac{A}{x} + \frac{C}{x^2 + 1} + \frac{D}{x^2}$
 d. $\frac{A}{x} + \frac{Bx^2 + C}{x^3}$

13. $\int_0^1 \tan^{-1}(x) dx$

- a. 1
 b. 0
 c. $\frac{\pi}{4}$
 d. $\frac{\pi}{4} - \frac{1}{2} \ln 2$

$u = \tan^{-1} x$ $du = \frac{1}{1+x^2} dx$
 $v = x$

$x \tan^{-1} x - \int \frac{x}{1+x^2}$

$x \tan^{-1} x - \frac{1}{2} \ln(1+x^2)$

$\tan^{-1} 1 - \tan^{-1}(0) - \left[\frac{1}{2} \ln 2 - \frac{1}{2} \ln 1 \right]$
 $= \frac{\pi}{4} - \frac{1}{2} \ln 2$

14. $\int_1^e \frac{\cos(\ln x)}{x} dx$

- a. diverge
 b. 1
 c. 0
 d. e

$u = \ln x$

$du = \frac{1}{x} dx$

$\int_0^{\pi/2} \cos u du = \sin u \Big|_0^{\pi/2} = 1$

15. $\int_0^1 \frac{x}{1+3x} dx$

a. $\frac{1}{3} - \frac{1}{3} \ln 4$
 b. $\frac{1}{3}$
 c. $\frac{1}{\sqrt{3}} \tan^{-1} \sqrt{3}$
 d. $\frac{1}{3} + \frac{1}{3} \ln 2$

$= \frac{1}{3} - \frac{1}{3} \ln 4$

16. $y = e^{\sinh x}$

a. $y' = \cosh x$

b. $y' = \cosh x e^{\sinh x}$

c. $y' = \sinh x e^{\sinh x}$

d. $y' = e^{\cosh x}$

17. the center of the ellipse

$$4x^2 + y^2 - 8x + 4y - 8 = 0$$

Is

a. (2,4)

b. (4,2)

c. (1,2)

d. (1,-2)

$$4(x^2 - 2x + 1) + (y^2 + 4y + 4) = 8 + 4 + 4$$

$$4(x-1)^2 + (y+2)^2 = 16$$

$$\frac{(x-1)^2}{4} + \frac{(y+2)^2}{16} = 1$$

18. the equation of the asymptotes for the hyperbola

$$4x^2 - 3y^2 + 8x + 16 = 0$$

a. $y = \frac{2}{\sqrt{3}}(x+1)$ and $y = -\frac{2}{\sqrt{3}}(x+1)$

b. $y = \frac{2}{\sqrt{3}}(x)$ and $y = -\frac{2}{\sqrt{3}}(x)$

c. $y-1 = \frac{2}{\sqrt{3}}(x)$ and $y-1 = -\frac{2}{\sqrt{3}}(x)$

d. $y = (x-1)$ and $y = -(x-1)$

$$\frac{y^2}{4} - \frac{(x+1)^2}{3} = 1$$

a=2
b=√3

19. the focus of the parabola $2y = 1 - x - x^2$ is

a. (0,0)

b. (-1, 1/2)

c. (1,-1)

d. (1, -1/2)

$$(x+1)^2 = -2(y-1/2)$$

$$(x-h)^2 = 4p(y-k)$$

$$(h,k) = (-1, 1/2)$$

$$p = -1/2$$

$$\text{Focus} = (-1, 1/2)$$

19) the focus of the parabola $2y = 1 - x - x^2$ is

- a. (0,0)
- b. $(-1, \frac{1}{2})$
- c. (1,-1)
- d. $(1, \frac{-1}{2})$

$$x^2 + x - 1 = -2y$$

$$x^2 + x + \frac{1}{4} = -2y + 1 + \frac{1}{4}$$

$$(x + \frac{1}{2})^2 = -2(y - \frac{5}{4})$$

20. Consider the function $f(x) = \frac{1}{x}$ find the maximum error in using a Taylor polynomial

Of order 3 centered at $a=1$ to estimate 1.2

- a. $(0.2)^4$
- b. $(1.2)^3$
- c. 1
- d. $(\frac{1}{1.2})^3$

$$R_3 = \frac{24(x-1)^4}{4! C^5} \quad 1 \leq C \leq 1.2$$

$$|R_3| \leq (1-1.2)^4 = (0.2)^4$$

21 From question 20 the infinite series represent

$$f(x) = \frac{1}{x} = \sum_{n=0}^{\infty} (1-x)^n$$

- a. true
- b. false

$$f(x) = \frac{1}{1-(1-x)} = \sum_{n=0}^{\infty} (1-x)^n$$

MATH DEPARTMENT

MATH 132 TEST THREE

TIME: 60 Min.

JANUARY 2008

NAME: George Hannonch

NUMBER: 106 15 15

SECTION: 1

Instructor's name: Dr. Rimon Jadin

82

28
30
10
14

QUESTION ONE: (MULTIPLE CHOICE) [40 POINTS]
CIRCLE THE RIGHT ANSWER:

1. Let $f(x) = \sum_{n=0}^{\infty} x^n$. The interval of convergence of the definite integral 0 to x,

$\int_0^x f(t) dt$ is

$$\frac{x^{n+1}}{n+1} \Big|_0^x = \frac{x^{n+1}}{n+1} - 0$$

$$\frac{x^{n+1} - x(x+1)}{x+1}$$

- (A) $x=0$ only
- (B) $|x| \leq 1$
- (C) $-\infty < x < \infty$
- (D) $-1 \leq x < 1$
- (E) $-1 < x < 1$

$$\frac{1}{2} \ln |1+x|$$

2. The coefficient of x^4 in the Maclaurin series for $f(x) = e^{-x/2}$ is

- (A) $-1/24$
- (B) $1/24$
- (C) $1/96$
- (D) $-1/384$
- (E) $1/384$

$$\frac{(-1)^n x^n}{n!} = \frac{(-1)^4 x^4}{4!} = \frac{1}{24}$$

$$\frac{1}{e^{x/2}}$$

$$\frac{1}{2!} \left(\frac{-x}{2}\right)^2$$

$$\frac{-1}{2e^{x/2}}$$

$$\frac{4}{4e^{x/2}}$$

(E)

3. Which of the following series diverges?

- (A) $\sum 1/n^2$
- (B) $\sum 1/(n^2 + n)$
- (C) $\sum n/(n^3 + 1)$

Con v
Con v.

(D) $\sum \frac{n}{\sqrt{4n^2 - 1}}$

$$\frac{n}{\sqrt{4n^2 - 1}} \sim \frac{1}{2}$$

(E) none of the preceding.

4. For which of the following series does the Ratio Test fail?

(A) $\sum 1/n!$

(B) $\sum n/2^n$

(C) $1 + 1/2^{3/2} + 1/3^{3/2} + 1/4^{3/2} + \dots$

(D) $(\ln 2)/2^2 + (\ln 3)/2^3 + (\ln 4)/2^4 + \dots$

(E) $\sum n^n/n!$

$(\frac{1}{2^n})^{3/2}$

$\frac{1}{3^{3/2}} \times 2^{3/2}$

$(\frac{2}{3})^{3/2}$ $(\frac{3}{4})^{3/2}$

$\frac{\ln 3}{2^3} \frac{2^4}{\ln 2}$

$\frac{1}{2} \frac{\ln 4}{\ln 3}$

5. Which of the following alternating series diverges?

(A) $\sum (-1)^{n-1}/n$

(B) $\sum (-1)^{n+1}(n-1)/(n+1)$

(C) $\sum (-1)^{n+1}/\ln(n+1)$

(D) $\sum \frac{(-1)^{n-1}}{\sqrt{n}}$

(E) $\sum (-1)^{n-1}n/n^2 + 1$

$\frac{2\sqrt{2n}}{1}$

6. Which of the following series converges conditionally?

(A) $3 - 1 + 1/9 - 1/27 + \dots$

(B) $\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} - \dots$

(C) $1/2^2 - 1/3^2 + 1/4^2 - \dots$

(D) $1 - 1.1 + 1.21 - 1.332 + \dots$

(E) $1/(1*2) - 1/(2*3) + 1/(3*4) - 1/(4*5) + \dots$

$\frac{1}{3}$ $\frac{1}{9}$ $\frac{1}{3}$

7. Suppose $f(x)$ is a function with Taylor series converging to $f(x)$ for all $x \in \mathbb{R}$.

If $f(0) = 2$, $f'(0) = 2$ and $f''(0) = 3$ for $n \geq 2$ then $f(x) =$

(A) $3e^x + 2x - 1$

(B) $e^{3x} + 2x + 1$

(C) $e^{3x} - x + 1$

(D) $3e^x - x - 1$

(E) $3e^x + 5x + 5$

$$\frac{x^n (1/n!)}{1/n!}$$

$$\frac{1/n!}{1/n!} = 1$$

8. Which of the following series converge?

(I) $\sum_{n=1}^{\infty} \frac{\ln(n^{-3})}{n^{-3}}$

(II) $\sum_{n=1}^{\infty} \frac{\ln 3}{3n}$

(III) $\sum_{n=1}^{\infty} \frac{n}{3^n}$

(A) I only

(B) II only

(C) III only

(D) None

(E) I and III

9. What is the Taylor series for $f(x) = e^x$ about $x = 1$?

(A) $\sum_{n=0}^{\infty} \frac{-(x-1)^n}{n!}$

(B) $\sum_{n=0}^{\infty} \frac{-e(x-1)^n}{n!}$

(C) $\sum_{n=0}^{\infty} \frac{(x-1)^n}{n!e}$

(D) $\sum_{n=0}^{\infty} \frac{e(x-1)^n}{n!}$

(E) $\sum_{n=0}^{\infty} \frac{(x-1)^n}{n!}$

10. Let $\{a_n\}$ be a sequence of positive real numbers such that

$\frac{1}{2} < \frac{a_{n+1}}{a_n} \leq \frac{n+4}{2n+1}$ for all n . Then $\lim_{n \rightarrow \infty} a_n =$

$\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} \leq \frac{1}{2}$

$\Rightarrow \sum a_n$ converges

$\Rightarrow \lim_{n \rightarrow \infty} a_n = 0$

(A) 0

(B) 1/2

(C) 1

(D) 2

(E) 4

QUESTION TWO: [30 points]

Test the following series for convergence or divergence. State the test you are using, and verify that it applies. Determine whether the convergence is absolute or conditional.

(a) $\sum_{n=3}^{\infty} \frac{1}{n (\ln n)^{100}}$ \Rightarrow Converges by integral test

$\int_3^{\infty} \frac{1}{n (\ln n)^{100}} = \frac{(\ln n)^{-99}}{-99} \Big|_3^{\infty}$

$= \frac{-1}{99 (\ln n)^{99}} \Big|_3^{\infty} = \frac{-1}{99} + \frac{1}{99 (\ln 3)^{99}} \Rightarrow \text{Integral Converges}$

$$(b) \sum_{n=0}^{\infty} \frac{n^{2n}}{(1+2n^2)^n} \Rightarrow \text{converges by the } n\text{th root test abs}$$

$$\lim_{n \rightarrow \infty} \sqrt[n]{\frac{n^{2n}}{(1+2n^2)^n}}$$

$$= \lim_{n \rightarrow \infty} \frac{n^2}{1+2n^2} = \boxed{\frac{1}{2}} < 1 \Rightarrow \text{converges}$$

$$(c) \sum_{n=1}^{\infty} \frac{n^3 - \sqrt{n} + 6}{n^4 - n - 3} \text{ diverges by limit comparison test}$$

$$\lim_{n \rightarrow \infty} \frac{n^3 - \sqrt{n} + 6}{n^4 - n - 3} \sim \frac{1}{n}$$

$$\lim_{n \rightarrow \infty} \frac{n^3 - \sqrt{n} + 6}{n^4 - n - 3} = \frac{1}{n}$$

$$= 1 \Rightarrow \text{both diverge or converge}$$

$$\frac{1}{n} \text{ diverges (power series with } p=1)$$

$$\Rightarrow \text{both diverge}$$

QUESTION THREE: [14 points]

Consider the power series $\sum_{n=1}^{\infty} \frac{(-2)^n}{\sqrt{n}} (x+3)^n$

- (a) When $x = -4$ does this series converge or diverge?
 (b) Determine all values for which the series converges.

$$\lim_{n \rightarrow \infty} \sqrt[n]{\frac{(-2)^n}{\sqrt{n}} (x+3)^n}$$

$$= \frac{-2}{\sqrt[n]{n}} (x+3) \quad \therefore |x+3| \leq \frac{1}{2} \Rightarrow R = \frac{1}{2}$$

$$-\frac{1}{2} < x+3 < \frac{1}{2}$$

$$-4 < x < -2$$

when $x = -4$

$$\sum \frac{(-2)^n (-1)^n}{\sqrt{n}}$$

$$\sum \frac{2^n}{\sqrt{n}}$$

$$\lim_{n \rightarrow \infty} \frac{2^n}{\sqrt{n}} \Rightarrow \frac{1}{\sqrt{n}}$$

$$\lim_{n \rightarrow \infty} 2^n = \infty$$

both diverge
by L.C.T

when $x = -2$

$$\sum \frac{(-2)^n}{\sqrt{n}}$$

converges conditionally by A.S.T

\Rightarrow the series converges on the interval $(-4, -2]$

QUESTION FOUR: [16 points]

Consider the integral $\int x \cos(x^3) dx$.

- (a) Write down the Maclaurin series for $\cos(x)$, $\cos(x^3)$, and $x \cos(x^3)$.
 (b) Evaluate $\int x \cos(x^3) dx$ as an infinite series.

$$\cos(x) = \frac{(-1)^n x^{2n}}{2n!}$$

$$\cos(x^3) = \frac{(-1)^n x^{6n}}{2n!}$$

Maclaurin

$$\cos(x^3) = 1 + \frac{x^6}{2!} + \frac{x^{12}}{4!} + \dots$$

$$\cos(x^3) = 1 + \frac{x^6}{2!} + \frac{x^{12}}{4!} + \frac{x^{18}}{6!} + \dots$$

$$x \cos(x^3) = x + \frac{x^7}{2!} + \frac{x^{13}}{4!} + \frac{x^{19}}{6!} + \dots$$

$$\int_0^1 x \cos(x^3) dx = x + \frac{x^7}{2!} + \frac{x^{13}}{4!} + \frac{x^{19}}{6!} + \frac{x^{25}}{8!} + \dots$$

$$\int_0^1 x \cos(x^3) dx = \frac{2x + x^7}{2} + \dots$$

$$\int_0^1 x \cos(x^3) dx = \frac{2}{2}$$

$$\lim_{x \rightarrow 0} \frac{\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots \right)}{\left(x + \frac{x^2}{2!} + \frac{x^3}{3!} \right)}$$

$$x \left(1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \dots \right)$$

$$\lim_{x \rightarrow 0} \frac{x \left(1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \dots \right)}{x \left(-1 + \frac{x}{2!} - \frac{x^2}{3!} + \dots \right)} = -1$$

جامعة بيرزيت

BIRZEIT UNIVERSITY

2017



2016

مجلس الطلبة

Question 3. (2 points) Express $\frac{1}{(1+x)^2}$ as a power series and find its radius of convergence.

(Hint: $\frac{1}{1+x} = 1 - x + x^2 - x^3 + \dots$)

~~$\frac{1}{x} = \sum_{n=0}^{\infty} x^n$~~

~~$\frac{1}{1-x} = \sum_{n=0}^{\infty} (-1)^n x^n$~~

~~$\frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n$~~

~~$\frac{1}{(1+x)^2} = \sum_{n=0}^{\infty} (-1)^n x^n$~~

$\frac{d}{dx} \left(\frac{1}{1+x} \right) = 1 - x + x^2 - x^3 + \dots$

$\frac{1}{(1+x)^2} = -1 + 2x - 3x^2 + \dots$

Question 4. (2 points) Use series to find $\lim_{x \rightarrow 0} \frac{\sin x}{e^{-x} - 1}$

$\sin x = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$

$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \Rightarrow e^{-x} = \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \dots$

$= 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \dots$

$\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots \right) / \left(1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \dots \right)$

$\left(1 - x + \frac{x^2}{2!} - \dots \right)$

BONUS. (2 points) Find the radius of convergence of $\sum_{n=1}^{\infty} \frac{x^n n!}{(1)(3)(5)\dots(2n-1)}$

~~$\frac{1}{1+x^2} = \sum_{n=0}^{\infty} (-1)^n (nx)^{n-1}$~~

~~$\frac{1}{1+x^2} = \sum_{n=1}^{\infty} (nx)^{n-1}$~~

by Ratio Test

$\lim_{n \rightarrow \infty} \left| \frac{((n+1)x)^n}{(nx)^{n-1}} \right| \Rightarrow \left| \frac{(n+1)^n x^n}{n^{n-1} x^{n-1}} \right|$

$|x| \lim_{n \rightarrow \infty} \frac{(n+1)^n}{n^{n-1}}$

11. The Taylor polynomial of order 3 generated by $f(x) = e^{2x}$ about $a = 0$ is

- a) $P_3(x) = 1 + 2x + x^2$
- b) $P_3(x) = 1 + 2x + 2x^2 + \frac{4}{3}x^3$**
- c) $P_3(x) = 1 + x + 2x^2 + \frac{4}{3}x^3$
- d) $P_3(x) = 1 + x + x^2$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$e^{2x} = \sum_{n=0}^{\infty} \frac{(2x)^n}{n!}$$

$$\textcircled{0} \quad \textcircled{1} \quad \textcircled{2}$$

$$1 + 2x + \frac{4x^2}{2}$$

$$= 1 + 2x + 2x^2 + \frac{48x^3}{36}$$

$$\frac{(2n+1)!}{(n+1)!(n+1)!} \neq \frac{2n!}{n!n!}$$

$$\frac{1}{n^2} = \boxed{0} \quad \lim_{n \rightarrow \infty}$$

$$\frac{(2n+1)}{(n+1)(n+1)}$$

$$x - \frac{x^5}{5!} + \frac{x^5}{5!}$$

Question 2. (4 points) Given that $\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$

(a) Find the Maclaurin series of $\cos x^3$.

$$\cos x^3 = \sum_{n=0}^{\infty} \frac{(-1)^n (x^3)^{2n}}{(2n)!} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{6n}}{(2n)!}$$

(b) Use part (a) to estimate $\int_0^1 \cos x^3 dx$ with error less than 0.01

$$\int_0^1 \sum_{n=0}^{\infty} \frac{(-1)^n x^{6n}}{(2n)!} dx = \int_0^1 \left(1 - \frac{x^6}{2!} + \frac{x^{12}}{4!} - \frac{x^{18}}{6!} \right) dx$$

$$= \left[x - \frac{x^7}{7(2!)} + \frac{x^{13}}{13(4!)} - \frac{x^{19}}{19(6!)} \right]_0^1$$

$\cos 0 = 1$
 $-\sin 0 = 0$
 $-\cos 0 = -1$

6. The series $\sum_{n=2}^{\infty} \frac{\ln n}{\sqrt{n^5}}$

- a) Converges by limit comparison with $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n^5}}$
- b) Converges by direct comparison with $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n^5}}$
- c) Converges by limit comparison with $\sum_{n=2}^{\infty} \frac{1}{n^2}$
- d) Diverges by limit comparison with $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}}$

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$
 $\frac{\ln n}{\sqrt{n^5}}$

$-1 + x - \frac{x^3}{3!}$

$\frac{1}{n^0} = 1$
 $= 1$
 $\lim_{n \rightarrow \infty} 1 = 1$

7. The series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^p}$ converges conditionally if

- a) $0 < p < 1$
- b) $0 \leq p < 1$
- c) $0 < p \leq 1$
- d) $0 \leq p \leq 1$

$\lim_{n \rightarrow \infty} \frac{1}{n^p}$
 $p < 1$

$\lim_{n \rightarrow \infty}$

$\frac{\ln n}{\sqrt{n^5}}$
 $\frac{1}{\sqrt{n^5}}$

diverges

8. The series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}(\sqrt{n}+1)}$

- a) Converges by integral test
- b) Diverges by integral test
- c) Converges by nth term test
- d) None of the above

$\lim_{n \rightarrow \infty} \frac{\ln n}{\sqrt{n^5}}$

$\frac{\ln n}{\sqrt{n^5}}$

$= \infty$
 $\left(\frac{-5}{2}\right) \left(\frac{1}{2}\right) \left(\frac{3}{2}\right)$
 $\frac{5}{2} > 1$

9. The binomial series of $\sqrt{1+x}$ is

- a) $1 - \frac{x}{2} + \frac{3x^2}{8} - \frac{5x^3}{16} + \dots$
- b) $1 + \frac{x}{2} - \frac{3x^2}{8} + \frac{5x^3}{16} + \dots$
- c) $1 - \frac{x}{2} - \frac{3x^2}{8} - \frac{5x^3}{16} + \dots$
- d) $1 - \frac{x}{2} + \frac{3x^2}{8} - \frac{5x^3}{16} + \dots$

$(1+x)^{\frac{1}{2}}$
 $m = -\frac{1}{2}$

$\frac{1}{\sqrt{n^5}}$

$\frac{\ln n}{\sqrt{n^5}}$

$\sum_{n=0}^{\infty} \binom{m}{n} x^n$
 $\left(\frac{-1/2}{1}\right) x$
 $\left(\frac{-1/2}{2}\right) x^2$

10. The Maclaurin series generated by $x \sin x^2$ is

- a) $x^3 + \frac{x^7}{3!} - \frac{x^{11}}{5!} + \dots$
- b) $x^3 - \frac{x^7}{3!} + \frac{x^{11}}{5!} - \dots$
- c) $x + \frac{x^5}{2!} - \frac{x^9}{4!} + \dots$
- d) $x - \frac{x^5}{2!} + \frac{x^9}{4!} - \dots$

$\frac{(2, -\frac{1}{2})}{2!}$

$\lim_{n \rightarrow \infty} \frac{1}{\ln n} \Rightarrow 0$

$\frac{3}{4} x^2$
 $\frac{1}{2} \left(\frac{-1}{2} - 1\right)$
 $2!$

$\frac{3x^2}{4}$
 $-\frac{1}{2} \left(\frac{-3}{2}\right) x^2$

Question 1. (12 points) Circle the best answer.

1. The series $\sum_{n=1}^{\infty} \left(\frac{4n+3}{3n-2}\right)^n$

Root test = $\lim_{n \rightarrow \infty} \sqrt[n]{\left(\frac{4n+3}{3n-2}\right)^n}$

$\lim_{n \rightarrow \infty} \left(\frac{4n+3}{3n-2}\right)$

$\frac{4}{3} > 1$

- a) Converges by root test
- b) Diverges by root test
- c) Converges by integral test
- d) Diverges by alternating series test

2. If we approximate e^x by $1+x+\frac{x^2}{2!}$, then the error in estimating e^{-1} is

- a) less than $\frac{1}{2}$
- b) less than $\frac{1}{2e}$
- c) less than $\frac{1}{6}$
- d) less than $\frac{1}{e}$

$e^x = 1+x+\frac{x^2}{2!}$

$e^x = \frac{x^n}{n!}$

$x = -1$
 $a = 0$
 $n = 2$

$\left| \frac{f^{(n+1)}(c)(x-a)^{n+1}}{(n+1)!} \right| < \frac{M(x-a)^{n+1}}{(n+1)!}$

$a < c < x$
 $0 < c < -1$

3. The radius of convergence of the series $\sum_{n=0}^{\infty} (n+1)! (x-4)^n$ is

- a) $R = 0$
- b) $R = 1$
- c) $R = 4$
- d) $R = \infty$



$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

$\left| \frac{e^c (x)^5}{3!} \right| < \frac{e^{-1} (x)^5}{3!}$

4. The series $\sum_{n=1}^{\infty} \frac{\cos n\pi}{\sqrt{n}}$

- a) Converges absolutely
- b) Converges conditionally
- c) Diverges by alternating series test
- d) Diverges by nth term test

$-1 + \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}}$

$\lim_{n \rightarrow \infty}$

Ratio Test

$\left| \frac{(n+1+1)! (x-4)^{n+1}}{(n+1)! (x-4)^n} \right|$

$\lim_{n \rightarrow \infty}$

$\left| \frac{(n+2)(n+1)! (x-4)^{n+1} (x-4)}{(n+1)! (x-4)^n} \right|$

5. $1 + \pi + \frac{\pi^2}{2!} + \frac{\pi^3}{3!} + \dots =$

- a) 0
- b) -1
- c) e^π
- d) None of the above

$|x-4|$

$\frac{1}{\sqrt{n}}$

$a_n = \frac{1}{\sqrt{n}}$ $u_n > 0$ decreasing

$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n}} = 0$



MATHEMATICS DEPARTMENT
MATH132 - THIRD EXAM -
SUMMER 2013/2014

12/20

Name: [Redacted]

Number: [Redacted]

(For Question 1) Fill your answers in the tables below:

Page 1	
1	b ✓
2	d ✓
3	d ✓
4	a ✓
5	c ✓

Page 2	
6	a ✓
7	d ✓
8	d ✓
9	a ✓
10	b ✓

Page 3	
11	b ✓
12	a ✓

Instructions:

1. No Calculators.
2. Mobiles Off.
3. BZU ID On Your Desk.
4. No Cheating At All.
5. Time Limit: 60 Minutes.

5

Final Exam (+4)

Shadi Key

Name (.....)

INSTRUCTORS : RASEM + SHADI

NO:

PROBLEM 1: 10 MARKS

Find the four fourth roots of $-8+8\sqrt{3}i$

The modulus = $\sqrt{(-8)^2 + (8\sqrt{3})^2} = 16$

$\cos \theta = \frac{-8}{16} = -\frac{1}{2} \Rightarrow \theta = \cos^{-1}(-\frac{1}{2}) = 120 = \frac{2\pi}{3}$

$w_k = 16^{1/4} [\cos(\frac{120 + 2k(180)}{4}) + i \sin(\frac{120 + 2k(180)}{4})]$ $k=0,1,2,3$

$w_0 = 2 [\cos(30) + i \sin(30)] = 2 [\frac{\sqrt{3}}{2} + \frac{1}{2}i] = \sqrt{3} + i$

$w_1 = 2 [\cos(120) + i \sin(120)] = 2 [-\frac{1}{2} + i \frac{\sqrt{3}}{2}] = -1 + i\sqrt{3}$

$w_2 = 2 [\cos(210) + i \sin(210)] = 2 [-\frac{\sqrt{3}}{2} - i \frac{1}{2}] = -\sqrt{3} - i$

$w_3 = 2 [\cos(300) + i \sin(300)] = 2 [\frac{1}{2} - i \frac{\sqrt{3}}{2}] = 1 - i\sqrt{3}$

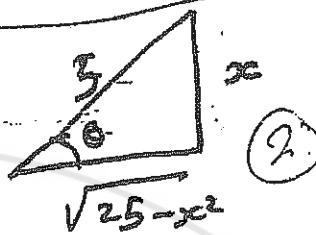
- $w_0 = \sqrt{3} + i$
- $w_1 = -1 + i\sqrt{3}$
- $w_2 = -\sqrt{3} - i$
- $w_3 = 1 - i\sqrt{3}$

PROBLEM 2: 10 MARKS EVALUATE

$$\int \frac{1}{x^2 \sqrt{25-x^2}} dx$$

$$\frac{x = 5 \sin \theta \quad (2)}{dx = 5 \cos \theta d\theta}$$

$$\int \frac{5 \cos \theta d\theta}{25 \sin^2 \theta \sqrt{25-25 \sin^2 \theta}} \quad (2)$$



$$\int \frac{5 \cos \theta d\theta}{25 \sin^2 \theta (5 \cos \theta)} \quad (2)$$

$$\frac{1}{25} \int \csc^2 \theta d\theta$$

$$= -\frac{1}{25} \cot(\theta) + C \quad (2)$$

$$= -\frac{1}{25} \frac{\sqrt{25-x^2}}{x} + C$$

AVI 505614 535

PROBLEM 3 (10 MARKS): CONSIDER s IS CONSTANT EVALUATE

$$\int_0^{\infty} t^2 e^{-st} dt$$

$$\begin{array}{l} t^2 \xrightarrow{-} e^{-st} \\ 2t \xrightarrow{-} -\frac{e^{-st}}{s} \\ 2 \xrightarrow{-} \frac{e^{-st}}{s^2} \\ 0 \xrightarrow{-} -\frac{e^{-st}}{s^3} \end{array}$$

$$\lim_{B \rightarrow \infty} \int_0^B t^2 e^{-st} dt$$

$$= \lim_{B \rightarrow \infty} \left[-\frac{t^2}{s} e^{-st} - \frac{2t}{s^2} e^{-st} - \frac{2}{s^3} e^{-st} \right]_0^B$$

$$= \lim_{B \rightarrow \infty} \left[-\frac{B^2}{s} e^{-sB} - \frac{2B}{s^2} e^{-sB} - \frac{2}{s^3} e^{-sB} - (0 - 0 - \frac{2}{s^3}) \right]$$

$$= \frac{2}{s^3}$$

PROBLEM 4 (10 MARKS) DISCUSS THE CONVERGENCE

$$\sum_{k=0}^{\infty} \frac{k^2}{2^{3k}} (x+4)^k$$

By Ratio test

$$\lim_{k \rightarrow \infty} \left| \frac{\frac{(k+1)^2}{2^{3(k+1)}} (x+4)^{k+1}}{\frac{k^2}{2^{3k}} (x+4)^k} \right| < 1$$

$$= \lim_{k \rightarrow \infty} \frac{(k+1)^2}{k^2} \left(\frac{1}{2^3} \right) |x+4| < 1$$

$$= \lim_{k \rightarrow \infty} \frac{1}{8} |x+4| < 1$$

$$\Rightarrow |x+4| < 8 \longrightarrow \boxed{R=8}$$

$$-8 < x+4 < 8$$

$$\boxed{-12 < x < 4}$$

when $\boxed{x = -12}$ $\sum_{k=0}^{\infty} \frac{k^2}{2^{3k}} (-12+4)^k = \sum_{k=0}^{\infty} \frac{k^2}{(8)^k} (-8)^k = \sum_{k=0}^{\infty} (-1)^k k^2$

$$\lim_{k \rightarrow \infty} k^2 = \infty \text{ div.}$$

when $\boxed{x = 4}$ $\sum_{k=0}^{\infty} \frac{k^2}{2^{3k}} (4+4)^k = \sum_{k=0}^{\infty} k^2$

$$\lim_{k \rightarrow \infty} k^2 = \infty \text{ div.}$$

The interval of conv. $(-12, 4)$

PROBLEM 5: 60 MARKS

Consider the test form the first column and the result form the second column

1. geometric series
2. p-series
3. telescoping series
4. the nth-term test
5. the integral test
6. alternating series test
7. the direct comparison test
8. the limit comparison test
9. the ratio test
10. the root test

- a. converges absolutely
- b. converges conditionally
- c. diverges

solve in details then circle the correct answer

1. radius of conv.

$$\sum_{k=1}^{\infty} \frac{2^k (x-3)^k}{\sqrt{k+3}} =$$

$$\lim \left| \frac{2^{k+1} (x-3)^{k+1}}{\sqrt{k+4}} \cdot \frac{\sqrt{k+3}}{2^k (x-3)^k} \right| < 1$$

- a. $\frac{1}{2}$
- b. $\frac{7}{2}$
- c. $\frac{19}{6}$
- d. $\frac{5}{2}$

$$= \lim 2|x-3| \frac{\sqrt{k+3}}{\sqrt{k+4}} < 1$$

$$= 2|x-3| < 1$$

$$|x-3| < \frac{1}{2}$$

$$-\frac{1}{2} < x-3 < \frac{1}{2}$$

$$\frac{5}{2} < x < \frac{7}{2}$$

$x = \frac{5}{2}$ by conv. by alternating

$x = \frac{7}{2}$ div. by limit comp. test

2. $\sum_{k=1}^{\infty} \frac{\sin k}{k^2 + 1}$

$a_k = \left| \frac{\sin k}{k^2 + 1} \right| \leq \frac{1}{k^2 + 1} \leq \frac{1}{k^2}$
 $\sum \frac{1}{k^2}$ conv. p-ser
 $\sum a_k$ conv. by D.C.T.

- a. divergent by p-series
- b. divergent by geometric series
- c. converges conditionally
- d. converges absolutely

3. $\sum_{k=0}^{\infty} \left(\frac{-1}{\sqrt{k+1}} \right)^k =$

- a. divergent by the ratio test
- b. divergent by the nth-root test
- c. converges conditionally
- d. converges absolutely

1) Let $a_k = \frac{1}{\sqrt{k+1}} > 0$
 2) $f(x) = \frac{1}{\sqrt{x+1}} = (x+1)^{-1/2}$
 $f'(x) = \frac{-1}{2\sqrt{(x+1)^3}} < 0$
 decreasing series

3) $\lim a_k = 0$
 $\sum (-1)^k a_k$ conv. alternating series

Let $b_k = \frac{1}{\sqrt{k}} = \frac{1}{k^{1/2}}$
 $\lim \frac{a_k}{b_k} = \lim \frac{\sqrt{k}}{\sqrt{k+1}} = 1$
 $\sum b_k$ div. p-series
 $\Rightarrow \sum a_k$ div. by limit C.T.

$$4. \sum_{k=2}^{\infty} \frac{(-1)^k \sqrt{k}}{\ln k} =$$

- a. divergent by the direct comparison test
b. divergent by the kth-term test
 c. converges absolutely
 d. converges conditionally

$$\lim_{k \rightarrow \infty} \frac{\sqrt{k}}{\ln k} = \lim_{k \rightarrow \infty} \frac{1}{2\sqrt{k}} = \lim_{k \rightarrow \infty} \frac{\sqrt{k}}{2} \neq 0$$

By k-th term test

the series div.

$$5. \sum_{k=1}^{\infty} \frac{\sqrt{n^2-1}}{n^3+2n^2+5} =$$

- a. convergent by the integral test
 b. divergent by limit comparison test
c. convergent by limit comparison test
 d. divergent by direct comparison test

$$\text{Let } a_n = \frac{\sqrt{n^2-1}}{n^3+2n^2+5}$$

$$b_n = \frac{1}{n^2}$$

$$\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = \frac{\sqrt{n^2-1} \cdot n^2}{n^3+2n^2+5} =$$

$$\text{But } \sum b_n = \sum \frac{1}{n^2}$$

Conv. p-series

So $\sum a_n$ Conv. Limit C.T.

$$6. \sum_{k=0}^{\infty} \frac{(-2)^{3k-1}}{9^k} = \sum_{k=0}^{\infty} \frac{(-2)^{-1} (-2)^{3k}}{9^k} = \sum_{k=0}^{\infty} -\frac{1}{2} \left(\frac{-8}{9}\right)^k$$

a. $\frac{-2}{9}$

b. $\frac{9}{7}$

c. $\frac{-9}{7}$

d. $\frac{-9}{34}$

$$= -\frac{1}{2} \left[\frac{1}{1 - (-\frac{8}{9})} \right]$$

$$= -\frac{1}{2} \left[\frac{1}{1 + \frac{8}{9}} \right]$$

$$= -\frac{1}{2} \left[\frac{9}{9+8} \right] = -\frac{9}{34}$$

$$7. \sum_{k=1}^{\infty} \frac{e^{1/n}}{2^n} =$$

$$\int_1^{\infty} \frac{e^{1/x}}{x^2} dx$$

$$u = \frac{1}{x} \quad | \quad x \rightarrow \infty \rightarrow u \rightarrow 0$$

$$du = -\frac{1}{x^2} dx$$

- a. convergent by the integral test**
 b. divergent by limit comparison test
 c. convergent by telescoping series
 d. divergent by direct comparison test

$$= \int_0^1 e^u du = e^u \Big|_0^1$$

$$= \int_0^1 e^u du = e^u \Big|_0^1$$

$$= e - 1$$

$e^{1/x}$ cont., +ve, dec.

$$8. \sum_{k=1}^{\infty} \frac{\cos\left(\frac{k\pi}{6}\right)}{k\sqrt{k}} =$$

$$a_k \leq \frac{|\cos k\pi|}{k\sqrt{k}} \leq \frac{1}{k^{3/2}} = b_k$$

$\sum b_k$ conv. p-series $\Rightarrow \sum a_k$ conv by D.C.T.

- a. divergent by the direct comparison test
 b. divergent by the kth-term test
 c. converges absolutely
 d. converges conditionally

$$9. \lim_{x \rightarrow 0} (\cos 3x)^{\frac{5}{x}} =$$

- a. 1
 b. 0
 c. $\frac{1}{2}$
 d. ∞

$$\text{Let } y = (\cos 3x)^{\frac{5}{x}} \Rightarrow \ln y = \frac{5 \ln(\cos 3x)}{x}$$

$$\ln y = \frac{5 \cos 3x}{x}$$

$$\lim_{x \rightarrow 0} \ln y = \lim_{x \rightarrow 0} \frac{5 \ln(\cos 3x)}{x} = \lim_{x \rightarrow 0} \frac{-15 \tan 3x}{1} = 0$$

$$\lim y = \lim e^{\ln y} = e^0 = 1$$

$$10. \lim_{x \rightarrow 0} (1-2x)^{\frac{1}{x}} =$$

- a. does not exit
 b. e^{-2}
 c. e^{-2}
 d. $\frac{2}{e}$

$$\text{Let } y = (1-2x)^{\frac{1}{x}}$$

$$\ln y = \frac{\ln(1-2x)}{x}$$

$$\lim_{x \rightarrow 0} \frac{\ln(1-2x)}{x}$$

$$\lim_{x \rightarrow 0} \frac{-2}{1-2x} = -2$$

$$\Rightarrow \lim (1-2x)^{\frac{1}{x}} = \lim e^y = e^{-2}$$

$$11. \text{ the sequence } a_n = \frac{\ln(2+e^n)}{3n}$$

- a. converges to $\frac{2}{3}$
 b. converges to $\frac{1}{3}$
 c. converges to 0
 d. divergent sequences

$$\lim \frac{\ln(2+e^n)}{3n}$$

$$\lim \frac{e^n}{\frac{2+e^n}{3}} = \lim \frac{e^n}{\frac{2}{3} + e^n}$$

$$= \lim \frac{e^n}{3e^n} = \frac{1}{3}$$

12. $\int \frac{2x^2 - 3x + 2}{x^3 + x}$ CAN BE INTEGRATED BY PARTIAL FRACTION

- a. $\frac{A}{X} + \frac{BX + C}{X^2 + 1}$
 b. $\frac{A}{X} + \frac{B}{X^2 + 1}$
 c. $\frac{A}{X} + \frac{C}{X^2 + 1} + \frac{D}{X^2}$
 d. $\frac{A}{X} + \frac{BX^2 + C}{X^3}$

13. $\int_0^1 \tan^{-1}(x) dx$

- a. 1
 b. 0
 c. $\frac{\pi}{4}$

d. $\frac{\pi}{4} - \frac{1}{2} \ln 2$

$u = \tan^{-1} x$ $dv = dx$
 $du = \frac{1}{1+x^2} dx$ $v = x$

$x \tan^{-1} x - \int \frac{x}{1+x^2}$

$x \tan^{-1} x - \frac{1}{2} \ln(1+x^2) \Big|_0^1$

$\tan^{-1} 1 - \tan^{-1}(0) - \left[\frac{1}{2} \ln 2 - \frac{1}{2} \ln 1 \right]$

$= \frac{\pi}{4} - \frac{1}{2} \ln 2$

14. $\int_1^{e^{\frac{\pi}{2}}} \frac{\cos(\ln x)}{x} dx$

- a. diverge
 b. 1
 c. 0
 d. e

$u = \ln x$

$du = \frac{1}{x} dx$

$\int_0^{\pi/2} \cos u du = \sin u \Big|_0^{\pi/2} = 1$

15. $\int_0^1 \frac{x}{1+3x} dx$

a. $\frac{1}{3} - \frac{1}{3} \ln 4$
 b. $\frac{1}{3}$
 c. $\frac{1}{\sqrt{3}} \tan^{-1} \sqrt{3}$
 d. $\frac{1}{3} + \frac{1}{3} \ln 2$

$= \frac{1}{3} - \frac{1}{3} \ln 4$

16. $y = e^{\sinh x}$

a. $y' = \cosh x$

b. $y' = \cosh x e^{\sinh x}$

c. $y' = \sinh x e^{\sinh x}$

d. $y' = e^{\cosh x}$

17. the center of the ellipse

$$4x^2 + y^2 - 8x + 4y - 8 = 0$$

Is

a. (2,4)

b. (4,2)

c. (1,2)

d. (1,-2)

$$4(x^2 - 2x + 1) + (y^2 + 4y + 4) = 8 + 4 + 4$$

$$4(x-1)^2 + (y+2)^2 = 16$$

$$\frac{(x-1)^2}{4} + \frac{(y+2)^2}{16} = 1$$

18. the equation of the asymptotes for the hyperbola

$$4x^2 - 3y^2 + 8x + 16 = 0$$

a. $y = \frac{2}{\sqrt{3}}(x+1)$ and $y = -\frac{2}{\sqrt{3}}(x+1)$

b. $y = \frac{2}{\sqrt{3}}(x)$ and $y = -\frac{2}{\sqrt{3}}(x)$

c. $y-1 = \frac{2}{\sqrt{3}}(x)$ and $y-1 = -\frac{2}{\sqrt{3}}(x)$

d. $y = (x-1)$ and $y = -(x-1)$

$$\frac{y^2}{4} - \frac{(x+1)^2}{3} = 1$$

$a = 2$
 $b = \sqrt{3}$

19. the focus of the parabola $2y = 1 - x - x^2$ is

a. (0,0)

b. $(-1, \frac{1}{2})$

c. (1,-1)

d. $(1, -\frac{1}{2})$

$$(x+1)^2 = -2(y-1)$$

$$(x-h)^2 = 4p(y-k)$$

$$(h,k) = (-1,1)$$

$$p = -\frac{1}{2}$$

$$\text{Focus} = (-1, \frac{1}{2})$$

19) the focus of the parabola $2y = 1 - x - x^2$ is.

a. (0,0)

b. $(-1, \frac{1}{2})$

c. (1,-1)

d. $(1, \frac{-1}{2})$

$$x^2 + x - 1 = -2y$$

$$x^2 + x + \frac{1}{4} = -2y + 1 + \frac{1}{4}$$

$$(x + \frac{1}{2})^2 = -2(y - \frac{5}{4})$$

20. Consider the function $f(x) = \frac{1}{x}$ find the maximum error in using a Taylor polynomial

Of order 3 centered at $a=1$ to estimate 1.2

a. $(0.2)^4$

b. $(1.2)^3$

c. 1

d. $(\frac{1}{1.2})^3$

$$R_3 = \frac{24(x-1)^4}{4! C^5} \quad 1 \leq C \leq 1.2$$

$$|R_3| \leq (1.2)^4 = (0.2)^4$$

21

From question 20 the infinite series represent

$$f(x) = \frac{1}{x} = \sum_{n=0}^{\infty} (1-x)^n$$

a. true

b. false

$$f(x) = \frac{1}{1-(1-x)} = \sum_{n=0}^{\infty} (1-x)^n$$

Birzeit University
Mathematics Department
Math 132
Final Exam
First Summer Semester 2012/2013

Student Name:

Student Number:

Time: 150 minutes

There are 4 questions in 10 pages

Question 1. (60%) Circle the most correct answer:

(1) The volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$, $x = 1$, and the x -axis, about the y -axis, is:

(a) $\frac{3\pi}{5}$

(b) $\frac{\pi}{5}$

(c) $\frac{2\pi}{5}$

(d) $\frac{4\pi}{5}$

(2) $\sum_{n=2}^{\infty} (0.5)^{-n} =$

(a) 2

(b) 1

(c) $\frac{1}{2}$

(d) None of the above

(3) $\int_0^{\frac{\pi}{2}} \tan x \, dx =$

(a) 0

(b) -1

(c) ∞

(d) $-\infty$

(4) If y is the solution of the differential equation $\frac{dy}{dx} = 3x^2y + y$, $y(1) = e$, then $y(-1) =$

(a) -1

(b) -3

(c) e^{-1}

(d) e^{-3}

$$(5) \int_1^4 \frac{3\sqrt{x}}{2\sqrt{x}} dx =$$

(a) $\frac{6}{\ln 3}$

(b) $\frac{3}{\ln 3}$

(c) $\frac{78}{\ln 3}$

(d) $\frac{9}{\ln 3}$

(6) The volume of the solid whose base is the region enclosed between the curves $y = x^2$ and $y = x$, and whose cross sections perpendicular to the x -axis are equilateral triangles of height 4, is:

(a) $\frac{1}{2}$

(b) $\frac{1}{3}$

(c) $\frac{1}{6}$

(d) $\frac{1}{4}$

(7) If $a_n = n3^{\frac{1}{n}}$, $n \in \mathbb{N}$, then $\lim_{n \rightarrow \infty} a_n =$

(a) 1

(b) 0

(c) ∞

(d) $\ln 3$

$$(8) \sum_{n=2}^{\infty} \frac{2n-1}{n^2(n-1)^2} =$$

(a) -1

(b) 1

(c) $\frac{1}{4}$

(d) 2

(9) Assuming its convergence, find the limit of the following recursively defined sequence, $a_1 = 8$,
 $a_{n+1} = \sqrt{a_n + 8} - 2$:

(a) 1

(b) -4

(c) -2

(d) 8

(10) $\int e^{\sqrt{2x+1}} dx =$

(a) $2\sqrt{2x+1}e^{\sqrt{2x+1}} + C$

(b) $\frac{e^{\sqrt{2x+1}}}{2\sqrt{2x+1}} + C$

(c) $\sqrt{2x+1}e^{\sqrt{2x+1}} - e^{\sqrt{2x+1}} + C$

(d) $\sqrt{2x+1}e^{\sqrt{2x+1}} - \sqrt{2x+1} + C$

(11) If $\tanh x = \frac{1}{2}$, $x < 0$, then $\operatorname{sech} x =$

(a) $\frac{\sqrt{5}}{2}$

(b) $\frac{-\sqrt{5}}{2}$

(c) $\frac{\sqrt{3}}{2}$

(d) $\frac{-\sqrt{3}}{2}$

(12) Which one of the following functions is the fastest growing as $x \rightarrow \infty$:

(a) $e^{\frac{x}{2}}$

(b) $\ln(\ln x)$

(c) 3^x

(d) $4 + 2^x$

(13) The series $\sum_{n=0}^{\infty} \frac{3^n}{5^n + 2^n}$:

(a) Converges by direct comparison with $\sum_{n=0}^{\infty} \frac{3^n}{4^n}$

(b) Converges by direct comparison with $\sum_{n=0}^{\infty} \frac{1}{2^n}$

(c) Converges by direct comparison with $\sum_{n=0}^{\infty} \frac{3^n}{7^n}$

(d) Converges by summing its terms as a geometric series

(14) The series $\sum_{n=2}^{\infty} \frac{(n+1)\ln n}{\sqrt{n}}$:

(a) Converges by the integral test

(b) Converges by direct comparison with $\sum_{n=2}^{\infty} \frac{1}{n^2}$

(c) Diverges by the ratio test

(d) Diverges by the n th-term test

(15) If $a_n = \left(1 - \frac{2}{n}\right)^{\frac{n}{2}}$, $n \in \mathbb{N}$, then $\lim_{n \rightarrow \infty} a_n =$

- (a) e^{-2}
- (b) e^{-1}
- (c) e^{-4}
- (d) $e^{-\frac{1}{3}}$

(16) The series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n(n+2)(n+3)}}$:

- (a) Converges by direct comparison with $\sum_{n=1}^{\infty} \frac{1}{\sqrt[5]{n^4}}$
- (b) Converges by limit comparison with $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3}}$
- (c) Converges by direct comparison with $\sum_{n=1}^{\infty} \frac{1}{\sqrt[4]{n^3}}$
- (d) Diverges by the ratio test

(17) $i^{215} =$

- (a) i
- (b) $-i$
- (c) 1
- (d) -1

(18) The integral $\int_2^{\infty} \frac{dx}{\sqrt[3]{x}(\sqrt{x}-1)}$:

- (a) Converges by limit comparison with $\int_2^{\infty} \frac{dx}{\sqrt[3]{x}}$
- (b) Converges by limit comparison with $\int_2^{\infty} \frac{dx}{\sqrt{x}}$
- (c) Diverges by direct comparison with $\int_2^{\infty} \frac{dx}{\sqrt[3]{x^2}}$
- (d) Diverges by direct comparison with $\int_2^{\infty} \frac{dx}{\sqrt[3]{x^5}}$

(19) The radius of convergence of the series $\sum_{n=1}^{\infty} \frac{e^n (x-1)^n}{n^2 3^n}$ is:

- (a) $\frac{3}{e} + 1$
- (b) $\frac{e}{3} + 1$
- (c) $\frac{3}{e}$
- (d) $\frac{e}{3}$

(20) $\int_0^1 x^2 \ln x \, dx =$

- (a) $-\frac{1}{4}$
- (b) $-\frac{1}{9}$
- (c) ∞
- (d) $-\infty$

(21) The series $\sum_{n=1}^{\infty} \frac{(-1)^n (\sqrt{n} + n)}{\sqrt{n^5 + 1}}$:

- (a) Converges absolutely
- (b) Converges conditionally
- (c) Diverges by limit comparison with $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3}}$
- (d) Diverges by the n th-term test

(22) $\int_1^{\sqrt{3}} \frac{dx}{x\sqrt{x^2+1}} =$

- (a) $\ln \left(\frac{\sqrt{3} + 1}{\sqrt{2}} \right)$
- (b) $\ln \left(\frac{\sqrt{2}}{\sqrt{3} + 1} \right)$
- (c) $\ln \left(\frac{\sqrt{2} + 1}{\sqrt{3}} \right)$
- (d) $\ln \left(\frac{\sqrt{3}}{\sqrt{2} + 1} \right)$

$$(23) \int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \sec^2 x \tan^2 x \, dx =$$

(a) $\frac{-26}{9\sqrt{3}}$

(b) $\frac{28}{9\sqrt{3}}$

(c) $\frac{-13}{3\sqrt{3}}$

(d) $\frac{20}{3\sqrt{3}}$

(24) A partial fraction for the function $f(x) = \frac{3x+1}{x^3-8}$ is:

(a) $\frac{A}{x-2} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$

(b) $\frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{x+2}$

(c) $\frac{A}{x-2} + \frac{Bx+C}{x^2-2x+4}$

(d) $\frac{A}{x-2} + \frac{Bx+C}{x^2+2x+4}$

(25) The series $\sum_{n=2}^{\infty} \left(\frac{n}{n^2-1}\right)^{n^2}$:

(a) Converges by summing its terms as a telescoping series

(b) Converges by the n th-term test

(c) Converges by the root test

(d) Diverges by direct comparison with $\sum_{n=2}^{\infty} \frac{1}{\sqrt[n]{n}}$

(26) $\frac{4-i}{1+i} =$

(a) $\frac{3}{2} - \frac{5}{2}i$

(b) $\frac{3}{2} + \frac{5}{2}i$

(c) $\frac{5}{2} - \frac{3}{2}i$

(d) $\frac{5}{2} + \frac{3}{2}i$

(27) The series $\sum_{n=3}^{\infty} \frac{\sqrt{\ln n}}{n^{1.1}}$:

(a) Converges by limit comparison with $\sum_{n=3}^{\infty} \frac{1}{n^{1.05}}$

(b) Converges by limit comparison with $\sum_{n=3}^{\infty} \frac{1}{n^{1.1}}$

(c) Converges by direct comparison with $\sum_{n=3}^{\infty} \frac{1}{n^{0.95}}$

(d) Diverges by the ratio test

(28) If $x = \ln(\sec t + \tan t)$, $y = t \sec t$, $-\frac{\pi}{2} < t < \frac{\pi}{2}$, then $\left. \frac{d^2y}{dx^2} \right|_{t=\frac{\pi}{4}} =$

(a) $\frac{\pi}{2} + 1$

(b) $\frac{\pi}{2\sqrt{2}} + \frac{1}{\sqrt{2}}$

(c) $\frac{\pi}{4} + 1$

(d) None of the above

(29) The series $\sum_{n=1}^{\infty} (-1)^n (\sqrt{n+1} - \sqrt{n-1})$:

(a) Converges absolutely

(b) Converges conditionally

(c) Diverges by the n th-term test

(d) Diverges by direct comparison with $\sum_{n=1}^{\infty} (-1)^n \sqrt{n}$

(30) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(n+1)2^{n+1}} =$

(a) $\ln\left(\frac{2}{3}\right)$

(b) $\ln\left(\frac{1}{3}\right)$

(c) $\ln\left(\frac{3}{2}\right)$

(d) $\ln\left(\frac{3}{4}\right)$

Question 2. (15%) (a) Use the binomial series to find out the first four nonzero terms of the Maclaurin series of $(1+x)^{\frac{2}{3}}$, $-1 < x < 1$.

(b) (1) Find the Taylor series of $f(x) = \tan^{-1}(3x^2)$, about $a = 0$, and specify its interval of convergence.

2017

2016

(2) Use the above series to estimate the value of $\tan^{-1}\left(\frac{1}{3}\right)$ with an error of magnitude less than 0.001.

Question 3. (13%) (a) Find the length of the parametric curve:

$$x = t, \quad y = \frac{t^2}{2}, \quad 0 \leq t \leq 1.$$

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(b) Sketch the parametric curve defined by the equations:

$$x = 3 \cos t, \quad y = 2 \sin t, \quad -\frac{\pi}{2} \leq t \leq \pi.$$

مجلس الطلبة

Question 4. (12%) (a) Find the four fourth roots of -81 .

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(b) Solve the equation: $2|z - 1 - i| = |z + \bar{z} - 2|$.

2017 2016

مجلس الطلبة

Birzeit University- Mathematics Department
Calculus II-Math 132

Final Exam

Spring 2013/2014

Name(Arabic):.....

Number:.....

Instructor of Discussion(Arabic):.....

Section:.....

Question 1.(90.5%) Solve the following then circle the correct answer:

1. The series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n(n+1)}$

- (a) Diverges by alternating series test.
- (b) Converges conditionally.
- (c) Converges absolutely.
- (d) Diverges by integral test.

2. $\int_1^e \frac{\ln x}{x^2} dx =$

- (a) $1 - 2e^{-1}$.
- (b) $\frac{1}{2}$.
- (c) $-e^{-1} \ln 2$.
- (d) $4e^{-1} + 1$.

3. The first four terms of the binomial series of $(1+x)^{1/3}$ are

- (a) $1 + x - \frac{1}{3}x^2 + \frac{1}{81}x^3$.
- (b) $1 + \frac{1}{8}x + \frac{1}{9}x^2 + \frac{2}{81}x^3$.
- (c) $1 + \frac{1}{3}x + \frac{1}{4}x^2 + \frac{1}{5}x^3$.
- (d) $1 + \frac{1}{3}x - \frac{1}{9}x^2 + \frac{5}{81}x^3$.

4. The interval of convergence of the series $\sum_{n=1}^{\infty} \frac{(-1)^n 2^n x^n}{n}$ is

- (a) $[-1, 1]$.
- (b) $[-2, 2]$.
- (c) $(-\frac{1}{2}, \frac{1}{2}]$.
- (d) $[-\frac{1}{2}, \frac{1}{2})$.

5. The Maclaurin series generated by e^{x^2} is

- (a) $\sum_{n=0}^{\infty} \frac{x^{2n}}{n!}$.
- (b) $\sum_{n=0}^{\infty} \frac{x^n}{n!}$.
- (c) $\sum_{n=0}^{\infty} \frac{x^n}{(2n)!}$.
- (d) $\sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}$.

6. $\int_1^2 x^3(\ln x) dx =$

- (a) $3 \ln 2 + \frac{15}{16}$.
- (b) $4 \ln 2 - \frac{15}{16}$.
- (c) $3 \ln -\frac{5}{6}$.
- (d) $3 \ln 2 + \frac{5}{6}$.

7. The series $\sum_{n=1}^{\infty} \frac{\sin^2 n}{n^{3/2}}$

- (a) Converges by direct comparison with $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$.
- (b) Diverges by n th term test.
- (c) Diverges by integral test.
- (d) Converges conditionally.

8. $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$ converges to

- (a) e^{-1} .
- (b) e .
- (c) 1.
- (d) Diverges.

9. $\sum_{n=0}^{\infty} \frac{2^{n+1}}{5^n} =$

- (a) $\frac{10}{3}$.
- (b) $\frac{5}{3}$.
- (c) 2.
- (d) $\frac{1}{3}$.

10. The sequence $a_n = \left(1 - \frac{2}{n}\right)^n$

- (a) Converges to 1.
- (b) Converges to e^{-2} .
- (c) Converges to -2 .
- (d) Diverges.

11. The slope of the parametric curve $x = 2 \cos t, y = 3 \sin t$ at $t = \frac{\pi}{4}$ is

- (a) $\frac{3}{2}$.
- (b) $\frac{2}{3}$.
- (c) $-\frac{3}{2}$.
- (d) $-\frac{2}{3}$.

12. If $x = 1 + \cos \theta, y = 2 + \sin \theta$ then $\frac{d^2y}{dx^2}$ when $\theta = \frac{\pi}{4}$ is

- (a) $-\sqrt{2}$.
- (b) -1 .
- (c) $-2\sqrt{2}$.
- (d) 2.

13. $\int_2^{\infty} \frac{dx}{x(\ln x)^{1/3}}$

- (a) Converges to $2 \ln 2$.
- (b) Converges to $\frac{1}{\ln 2}$.
- (c) Converges to $\frac{2}{\sqrt{\ln 2}}$.
- (d) Diverges.

14. The sum of the telescoping series $\sum_{n=2}^{\infty} \frac{2}{n^2-1}$ is

- (a) 1.
- (b) $\frac{1}{2}$.
- (c) $\frac{3}{2}$.
- (d) $\frac{2}{3}$.

15. $\int_0^{\pi/2} \cos^3 x \sqrt{\sin x} dx =$

- (a) $\frac{2}{3}$.
- (b) $\frac{20}{21}$.
- (c) $\frac{8}{21}$.
- (d) $\frac{2}{7}$.

16. The series $\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$

- (a) Diverges.
- (b) Converges conditionally.
- (c) Converges absolutely.
- (d) None of the above.

17. $\int_0^1 \sin^{-1} x dx =$

- (a) $\frac{\pi}{2} - 1$.
- (b) $\frac{\pi}{2}$.
- (c) 1.
- (d) $\frac{\pi}{2} + 1$.

18. $\int_1^2 \frac{dx}{x(x+1)} =$

- (a) $\ln 4 - \ln 3$.
- (b) $-\ln 2$.
- (c) $\ln 2 - \ln 3$.
- (d) $\frac{1}{6}$.

19. $4^{2 \log_4 2} =$

- (a) 1.
- (b) 4.
- (c) $\ln 4$.
- (d) 2.

20. The length of the curve $x = t, y = \cosh t, 0 \leq t \leq 1$ is

- (a) $\sinh 1$.
- (b) $\cosh 1$.
- (c) 1.
- (d) e .

21. The series $\sum_{n=1}^{\infty} \frac{(\ln n)^2}{n^2}$

- (a) Converges by limit comparison test with $\sum_{n=1}^{\infty} \frac{1}{n^2}$.
- (b) Converges by limit comparison test with $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$.
- (c) Diverges by limit comparison test with $\sum_{n=1}^{\infty} \frac{1}{n}$.
- (d) Diverges by limit comparison test with $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$.

22. The series $\sum_{n=1}^{\infty} \ln \left(1 + \frac{1}{n}\right)$

- (a) Diverges by nth term test.
- (b) Converges by integral test.
- (c) Diverges by direct comparison with $\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)$.
- (d) Diverges by limit comparison with $\sum_{n=1}^{\infty} \frac{1}{n}$.

23. The sequence $a_n = \ln(n+2) - \ln(n+1)$

- (a) Converges to 1.
- (b) Converges to 0.
- (c) Converges to e^{-1} .
- (d) Diverges.

24. The sequence $a_n = n \sin\left(\frac{\pi}{n}\right)$

- (a) Converges to π .
- (b) Converges to 0.
- (c) Converges to 1.
- (d) Diverges.

25. Using alternating series estimation, we can approximate $\cos x$ by $1 - \frac{x^2}{2}$ with error less than 0.01 if

- (a) $|x| < 0.01$.
- (b) $|x| < \sqrt{(4!)(0.01)}$.
- (c) $|x| < \sqrt[4]{(4!)(0.01)}$.
- (d) $|x| < \sqrt[4]{(0.01)}$.

26. The series $\sum_{n=1}^{\infty} \ln\left(\frac{n}{2n+1}\right)$

- (a) Converges to 0.
- (b) Converges to $\ln(1/2)$.
- (c) Diverges by nth term test.
- (d) Diverges by alternating series test.

27. $\sum_{n=1}^{\infty} \frac{2^n}{n!}$

- (a) Converges by ratio test.
- (b) Diverges by ratio test.
- (c) Diverges by root test.
- (d) Converges by direct comparison test with $\sum_{n=1}^{\infty} \frac{1}{n!}$.

28. $\left(\frac{1+i}{1-i}\right)^2 =$

- (a) 1.
- (b) i .
- (c) $-i$.
- (d) -1.

29. The functions $x^2, e^x, 10^x, \ln x$ from slowest to fastest are

- (a) $\ln x, x^2, 10^x, e^x$.
- (b) $x^2, \ln x, e^x, 10^x$.
- (c) $x^2, \ln x, 10^x, e^x$.
- (d) $\ln x, x^2, e^x, 10^x$.

30. The radius of convergence of the series $\sum_{n=1}^{\infty} \frac{2^n(x-2)^n}{n}$ is

- (a) 1.
- (b) $\frac{1}{2}$.
- (c) 4.
- (d) 2.

31. The error in the estimation $\sqrt{1+x} \approx 1 + \frac{1}{2}x$ in the interval $[0, 0.1]$ using Taylor's theorem is less than

- (a) 8×10^{-2} .
- (b) 4×10^{-2} .
- (c) $\frac{1}{8} \times 10^{-2}$.
- (d) None of the above.

32. One of the following is false

- (a) The series $\sum_{n=1}^{\infty} \frac{n}{n+1}$ diverges by nth term test.
- (b) The series $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$ converges by integral test.
- (c) The series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{5/4}}$ converges conditionally.
- (d) The series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^4+1}}$ converges by limit comparison with $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

33. $\int_0^1 \frac{dx}{\sqrt{1+x^2}} =$

- (a) $\ln \sqrt{2}$.
- (b) $\ln(1 + \sqrt{2})$.
- (c) $\ln 2$
- (d) None of the above.

34. $\int_1^e 4 \cosh(\ln x) dx =$

- (a) e .
- (b) e^2 .
- (c) $1 + e$.
- (d) $1 + e^2$.

35. $\int_1^e \frac{\log_4 t}{t} dt =$

- (a) $\ln 4$.
- (b) $\frac{1}{2 \ln 4}$.
- (c) $\frac{1}{4}$.
- (d) None of the above.

36. The integral $\int_2^\infty \frac{dx}{x + \ln x}$

- (a) Converges to 1.
- (b) Diverges by direct comparison with $\int_2^\infty \frac{dx}{x}$.
- (c) Diverges by limit comparison with $\int_2^\infty \frac{dx}{x}$.
- (d) Diverges by direct comparison with $\int_2^\infty \frac{dx}{\ln x}$.

37. The parametric curve $x = \cos^2 t, y = \sin^2 t, 0 \leq t \leq \frac{\pi}{2}$ presents

- (a) The line segment from $(0, 1)$ to $(1, 0)$.
- (b) The line segment from $(0, 1)$ to $(-1, 0)$.
- (c) The line segment from $(1, 0)$ to $(0, 1)$.
- (d) The unit circle.

Question 2(8%) Find the six sixth roots of $-i$. Write all roots in the form of $a + ib$.





BIRZEIT UNIVERSITY

MATHEMATICS DEPARTMENT
MATH132 - THIRD EXAM
SUMMER 2013/2014

Name... ~~XXXXXXXXXX~~.....

Number... ~~XXXXXXXXXX~~.....

(For Question 1) Fill your answers in the tables below:

Page 1	
1	b ✓
2	d ✓
3	d ✓
4	a ✓
5	c ✓

Page 2	
6	a ✓
7	d ✓
8	d ✓
9	a ✓
10	b ✓

Page 3	
11	b ✓
12	a ✓

Instructions:

1. No Calculators.
2. Mobiles Off.
3. BZU ID On Your Desk.
4. No Cheating At All.
5. Time Limit: 60 Minutes.

Question 1. (12 points) Circle the best answer.

1. The series $\sum_{n=1}^{\infty} \left(\frac{4n+3}{3n-2}\right)^n$

- a) Converges by root test
- b) Diverges by root test
- c) Converges by integral test
- d) Diverges by alternating series test

Root test = $\lim_{n \rightarrow \infty} \sqrt[n]{\frac{4n+3}{3n-2}}$

$$\lim_{n \rightarrow \infty} \left(\frac{4n+3}{3n-2}\right)^{\frac{1}{n}}$$

$$\frac{4}{3} > 1$$

2. If we approximate e^x by $1+x+\frac{x^2}{2!}$, then the error in estimating e^{-1} is

- a) less than $\frac{1}{2}$
- b) less than $\frac{1}{2e}$
- c) less than $\frac{1}{6}$
- d) less than $\frac{1}{e}$

$$e^x = 1 + x + \frac{x^2}{2!}$$

$$e^x = \frac{x^n}{n!}$$

$$\begin{aligned} X &= -1 \\ \bar{a} &= 0 \\ n &= 2 \end{aligned}$$

$$\left| \frac{f^{(n+1)}(c)(x-a)^{n+1}}{(n+1)!} \right| < \frac{M(x-a)^{n+1}}{(n+1)!}$$

$$\begin{aligned} a < c < x \\ 0 < c < -1 \end{aligned}$$

3. The radius of convergence of the series $\sum_{n=0}^{\infty} (n+1)! (x-4)^n$ is

- a) $R=0$
- b) $R=1$
- c) $R=4$
- d) $R=\infty$



$$= \sum_{n=1}^{\infty} \frac{(n!)^n}{\sqrt{n}}$$

$$-1 + \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}}$$

$$\left| \frac{e^c (x)^5}{5!} \right| < \frac{e^{-1} (x)^5}{5!}$$

4. The series $\sum_{n=1}^{\infty} \frac{\cos n\pi}{\sqrt{n}}$

- a) Converges absolutely
- b) Converges conditionally
- c) Diverges by alternating series test
- d) Diverges by nth term test

Ratio's test

$$\lim_{n \rightarrow \infty} \frac{(n+1+1)! (x-4)^{n+1}}{(n+1)! (x-4)^n}$$

$$\lim_{n \rightarrow \infty} \frac{(n+2)(n+1)! (x-4)^{n+1} (x-4)^n}{(n+1)! (x-4)^{2n}}$$

5. $1 + \pi + \frac{\pi^2}{2!} + \frac{\pi^3}{3!} + \dots =$

- a) 0
- b) -1
- c) e^π
- d) None of the above

$$|x-4|$$

$$\frac{1}{\sqrt{n}}$$

$$-a_n = \frac{1}{\sqrt{n}} \quad \text{un} > 0 \text{ decreasing}$$

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n}} = 0$$

$\cos 0 = 1$
 $-\sin 0 = 0$
 $-\cos 0 = -1$

6. The series $\sum_{n=2}^{\infty} \frac{\ln n}{\sqrt{n^5}}$

- a) Converges by limit comparison with $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n^5}}$
- b) Converges by direct comparison with $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n^5}}$
- c) Converges by limit comparison with $\sum_{n=2}^{\infty} \frac{1}{n^2}$
- d) Diverges by limit comparison with $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}}$

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$
 $\frac{\ln n}{\sqrt{n^5}}$
 $\frac{1}{n^0} = 1$
 $= 1$
 $\lim_{n \rightarrow \infty} 1 = 1$
 $1 \neq 0$
 diverges

7. The series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^p}$ converges conditionally if

- a) $0 < p < 1$
- b) $0 \leq p < 1$
- c) $0 < p \leq 1$
- d) $0 \leq p \leq 1$

$\lim_{n \rightarrow \infty} \frac{\ln n}{\sqrt{n^5}}$
 $\frac{1}{n^p}$
 $\frac{1}{n^5}$
 $\frac{1}{2!}$
 $\frac{1}{2!}$
 $\frac{1}{2!}$

8. The series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}(\sqrt{n+1})}$

- a) Converges by integral test
- b) Diverges by integral test
- c) Converges by nth term test
- d) None of the above

$\lim_{n \rightarrow \infty} \frac{\ln n}{\sqrt{n^5}} = \infty$
 $\frac{1}{n^{\frac{p}{2}}}$
 $\frac{5}{2} > 1$
 Converges

9. The binomial series of $\frac{1}{\sqrt{1+x}}$ is

- a) $1 - \frac{x}{2} + \frac{3x^2}{8} - \frac{5x^3}{16} + \dots$
- b) $1 + \frac{x}{2} - \frac{3x^2}{8} + \frac{5x^3}{16} + \dots$
- c) $1 - \frac{x}{2} - \frac{3x^2}{8} - \frac{5x^3}{16} - \dots$
- d) $1 - \frac{x}{2} + \frac{3x^2}{8} - \frac{5x^3}{16} + \dots$

$\frac{1}{\sqrt{n^5}}$
 $\frac{\ln n}{\sqrt{n^5}}$
 $\frac{1}{\sqrt{n^5}}$
 $(\frac{m}{n}) X^n$
 $(-\frac{1}{2}) X$
 $(-\frac{1}{2}) X^2$

10. The Maclaurin series generated by $x \sin x^2$ is

- a) $x^3 + \frac{x^7}{3!} - \frac{x^{11}}{5!} + \dots$
- b) $x^3 - \frac{x^7}{3!} + \frac{x^{11}}{5!} - \dots$
- c) $x + \frac{x^5}{2!} - \frac{x^9}{4!} + \dots$
- d) $x - \frac{x^5}{2!} + \frac{x^9}{4!} - \dots$

$\frac{1}{\sqrt{n^5}}$
 $\frac{\ln n}{\sqrt{n^5}}$
 $\lim_{n \rightarrow \infty} \frac{\ln n}{\sqrt{n^5}} \Rightarrow 0$
 $\frac{1}{2}$
 $\frac{1}{2}$
 $\frac{1}{2}$

$\frac{1}{4} x^2$
 $\frac{1}{2} x^2$
 $\frac{1}{2!} (-\frac{1}{2} - 1)$
 $\frac{1}{2!}$

11. The Taylor polynomial of order 3 generated by $f(x) = e^{2x}$ about $a = 0$ is

- a) $P_3(x) = 1 + 2x + x^2$
- b) $P_3(x) = 1 + 2x + 2x^2 + \frac{4}{3}x^3$**
- c) $P_3(x) = 1 + x + 2x^2 + \frac{4}{3}x^3$
- d) $P_3(x) = 1 + x + x^2$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$e^{2x} = \sum_{n=0}^{\infty} \frac{(2x)^n}{n!}$$

$$1 + 2x + \frac{4x^2}{2} + \frac{8x^3}{6}$$

12. The series $\sum_{n=1}^{\infty} \frac{(2n)!}{n!n!}$

- a) Converges by ratio test**
- b) Diverges by ratio test
- c) Converges to 4
- d) Converges by root test

$$\frac{(2(n+1))!}{(n+1)!(n+1)!} \div \frac{(2n)!}{n!n!} = \frac{(2n+1)(2n+2)}{(n+1)(n+1)} \cdot \frac{n!n!}{(2n)!}$$

$$\frac{1}{n^2} = \boxed{0} < 1 \quad \lim_{n \rightarrow \infty}$$

$$\frac{(2n+1)(2n+2)}{(n+1)(n+1)} \cdot \frac{n!n!}{(2n)!}$$

$$X - \frac{X^3}{3!} + \frac{X^5}{5!} - \frac{X^7}{7!} + \frac{X^9}{9!} - \frac{X^{11}}{11!} + \frac{X^{13}}{13!} - \frac{X^{15}}{15!} + \frac{X^{17}}{17!} - \frac{X^{19}}{19!} + \frac{X^{21}}{21!} - \frac{X^{23}}{23!} + \frac{X^{25}}{25!} - \frac{X^{27}}{27!} + \frac{X^{29}}{29!} - \frac{X^{31}}{31!} + \frac{X^{33}}{33!} - \frac{X^{35}}{35!} + \frac{X^{37}}{37!} - \frac{X^{39}}{39!} + \frac{X^{41}}{41!} - \frac{X^{43}}{43!} + \frac{X^{45}}{45!} - \frac{X^{47}}{47!} + \frac{X^{49}}{49!} - \frac{X^{51}}{51!} + \frac{X^{53}}{53!} - \frac{X^{55}}{55!} + \frac{X^{57}}{57!} - \frac{X^{59}}{59!} + \frac{X^{61}}{61!} - \frac{X^{63}}{63!} + \frac{X^{65}}{65!} - \frac{X^{67}}{67!} + \frac{X^{69}}{69!} - \frac{X^{71}}{71!} + \frac{X^{73}}{73!} - \frac{X^{75}}{75!} + \frac{X^{77}}{77!} - \frac{X^{79}}{79!} + \frac{X^{81}}{81!} - \frac{X^{83}}{83!} + \frac{X^{85}}{85!} - \frac{X^{87}}{87!} + \frac{X^{89}}{89!} - \frac{X^{91}}{91!} + \frac{X^{93}}{93!} - \frac{X^{95}}{95!} + \frac{X^{97}}{97!} - \frac{X^{99}}{99!}$$

Question 2. (4 points) Given that $\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$

(a) Find the Maclaurin series of $\cos x^3$.

$$\cos x^3 = \sum_{n=0}^{\infty} \frac{(-1)^n (x^3)^{2n}}{(2n)!} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{6n}}{(2n)!}$$

(b) Use part (a) to estimate $\int_0^1 \cos x^3 dx$ with error less than 0.01

$$\int_0^1 \sum_{n=0}^{\infty} \frac{(-1)^n x^{6n}}{(2n)!} dx = \int_0^1 \left(1 - \frac{x^6}{2!} + \frac{x^{12}}{4!} - \frac{x^{18}}{6!} + \dots \right) dx$$

$$= \left[x - \frac{x^7}{7(2!)} + \frac{x^{13}}{13(4!)} - \frac{x^{19}}{19(6!)} + \dots \right]_0^1$$

$$\frac{1}{(1+x)^2}$$

Question 3. (2 points) Express $\frac{1}{(1+x)^2}$ as a power series and find its radius of convergence.

(Hint: $\frac{1}{1+x} = 1 - x + x^2 - x^3 + \dots$)

~~$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$~~ ~~$\frac{1}{1-x} = \sum_{n=0}^{\infty} (-1)^n x^n$~~

~~$\frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n$~~

~~$\frac{1}{(1+x)^2}$~~

$$\frac{d}{dx} \left(\frac{1}{1+x} \right) = -1 + x^2 - x^3 + \dots$$

$$\frac{1}{(1+x)^2} = -1 + 2x - 3x^2 + \dots$$

~~$$\frac{1}{1+x^2} = \sum_{n=0}^{\infty} (-1)^n (nx)^{n-1}$$~~

~~$$\frac{1}{1+x^2} = \sum_{n=1}^{\infty} (nx)^{n-1}$$~~

Question 4. (2 points) Use series to find $\lim_{x \rightarrow 0} \frac{\sin x}{e^{-x} - 1}$.

$$\sin x = \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \Rightarrow e^{-x} = \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!}$$

$$= 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots$$

~~$\frac{x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots}{1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots}$~~

~~$\frac{x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots}{1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots}$~~

by Ratio Test

$$\lim_{n \rightarrow \infty} \left| \frac{((n+1)x)^n}{(nx)^{n-1}} \right| \Rightarrow \left| \frac{(n+1)x^n}{n^{n-1} x^{n-1}} \right|$$

$$|x| \lim_{n \rightarrow \infty} \frac{(n+1)^n}{n^{n-1}}$$

BONUS. (2 points) Find the radius of convergence of $\sum_{n=1}^{\infty} \frac{x^n n!}{(1)(3)(5)\dots(2n-1)}$.

$$\lim_{x \rightarrow 0} \frac{\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots \right)}{\left(x + \frac{x^2}{2!} + \frac{x^3}{3!} \right)}$$

1 - 5

$$\lim_{x \rightarrow 0} \frac{x \left(1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \dots \right)}{x \left(1 + \frac{x}{2!} - \frac{x^2}{3!} - \dots \right)}$$

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2017



2016

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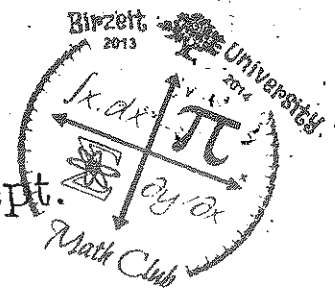
2017



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Birzeit University
 Math. & Comp. Science Dept.
 Math. 132



Dr. Marwan Awartani

Final Exam

Fall ~~2013~~

Student Name: _____ Number: _____ Section: _____

Q1: (60 points) Circle the MOST correct answer:

1. $\int_e^{e^2} \frac{1}{x \ln x} dx =$

- (a) 1
- (b) $\ln 2$
- (c) $\ln(\frac{1}{2})$
- (d) 0

$$u = \ln x$$

$$du = \frac{dx}{x} \quad dx = x du$$

$$= \int \frac{du}{u} = \ln|u| \Big|_e^{e^2} = \ln|\ln x| \Big|_e^{e^2}$$

$$= \ln \ln e^2 - \ln \ln e$$

$$= \ln 2$$

2. The curve with parametric equations $x = \sin t, y = \cos t, -\infty < t < \infty$.

- (a) A segment of a parabola
- (b) A circle
- (c) An ellipse
- (d) A hyperbola

3. The slope of the curve $x = \sin 2t, y = \cos t$ at $t = +\frac{\pi}{6}$ is:

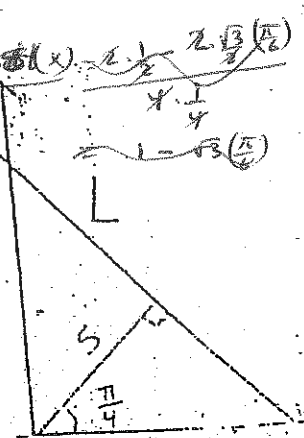
- (a) 0
- (b) 1
- (c) $\frac{1}{2}$
- (d) $-\frac{1}{2}$

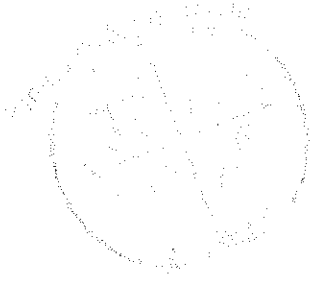
$x = 2 \sin t \cos t$
 $x = 2 \sin \frac{\pi}{6} \cos \frac{\pi}{6} = 2(\frac{\sqrt{3}}{2} \cdot \frac{1}{2}) = \frac{\sqrt{3}}{2}$

$y = \frac{x}{2 \sin t} \Rightarrow y = \frac{2 \sin t \cos t}{2 \sin t} = \cos t = \frac{1}{2}$
 $\frac{dy}{dx} = \frac{y}{x} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$

4. The polar equation of straight line l , is:

- (a) $r \cos(\theta + \frac{\pi}{4}) = -5$
- (b) $r \cos(\theta - \frac{\pi}{4}) = 5$
- (c) $r \sin(\theta + \frac{\pi}{4}) = -5$
- (d) $r \cos \theta = 5$





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2017



2016

مَجْلِسُ الطَّلَبَةِ

5. The graphs of the curves with polar coordinates $r = \sin \theta$, $r = -\cos \theta$ intersects at:

- (a) Only at the origin.
- (b) Only when $\tan \theta = -1$
- (c) At exactly two points.
- (d) At exactly three points.

6. The equation of $(x^2) + 5xy + y^2 = 3$ is

- (a) Circle
- (b) Ellipse
- (c) A hyperbola
- (d) A parabola

7. One of the following is not an improper integral

(a) $\int_0^{10} \frac{\sin x}{x} dx$

(b) $\int_{-\infty}^a x dx$

(c) $\int_0^1 \frac{1}{\sqrt{x}} dx$

(d) $\int_0^{\infty} \frac{dx}{x^2 - 1}$

8. $\int \sin^2 x \cos^3 x dx =$

(a) $\frac{\sin^3 x \cos^4 x}{12} + c$

(b) $\frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + c$

(c) $\frac{\cos^3 x \sin^4 x}{15} + c$

(d) $\frac{\sin^3 x}{15} (5 + 3 \sin^2 x) + c$

9. If a particle moves on a parametric curve described by $x = t^2, y = \sqrt{1-t^4}$, $-1 \leq t \leq 1$, then

- (a) The initial point is (1,0) and the end point is (0,1).
- (b) the motion is clockwise.
- (c) the motion is counter clockwise.
- (d) None of the above.

10. $\lim_{x \rightarrow 0^+} (1+x)^{\frac{1}{x}}$

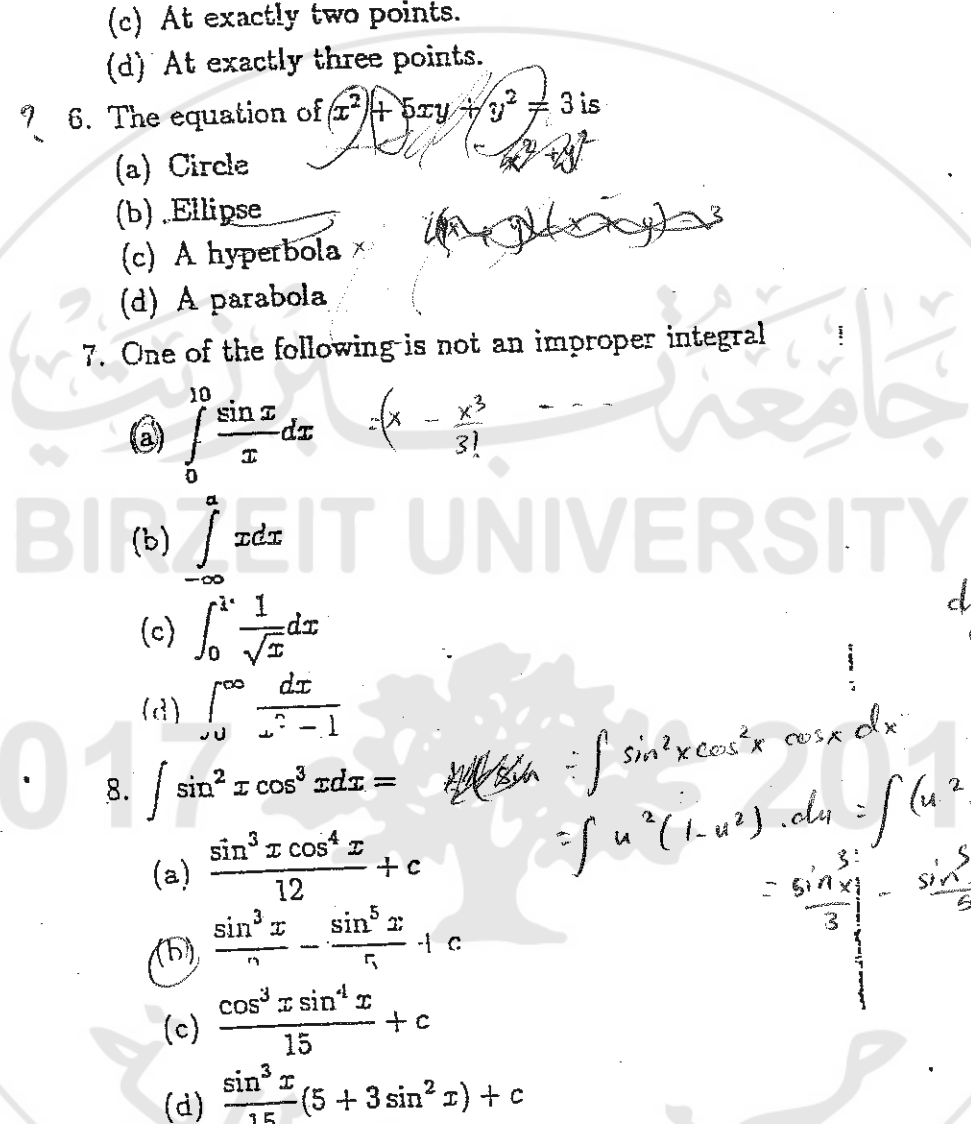
- (a) e
- (b) 1
- (c) $\frac{1}{e}$
- (d) 0

$\lim_{x \rightarrow \infty} (1 + \frac{1}{x})^x$

$u = \frac{1}{x}$
 $du = -\frac{dx}{x^2} \rightarrow dx = -x^2 du$
 $x \rightarrow 0 \text{ as } u \rightarrow \infty$

$du = \cos x dx$
 $u = \sin x$

$\int \sin^2 x \cos^3 x dx = \int \sin^2 x \cos^2 x \cos x dx$
 $= \int u^2 (1-u^2) du = \int (u^2 - u^4) du$
 $= \frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + c$



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2017



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10. The graph of the curves with polar coordinates $r = 1 + \cos \theta$ and $r = 1 - \cos \theta$ intersects in

- (a) One point only
- (b) Two points only
- (c) Three points only
- (d) Four points only

11. The slope of the polar curve $r = 1 + 2 \cos \theta$ at the origin is

- (a) 1
- (b) -1
- (c) $\sqrt{3}$
- (d) $\pm\sqrt{3}$

12. $\lim_{x \rightarrow 0^+} (1+x)^{\frac{1}{x}} =$

- (a) e
- (b) 1
- (c) $\frac{1}{e}$
- (d) 0

Handwritten solution for Q12:
 $u = \tan^{-1} x$
 $x = \tan u$
 $dx = \sec^2 u \cdot du$
 $\lim_{x \rightarrow 0} \frac{\tan^{-1} x}{x} = \lim_{u \rightarrow 0} \frac{u}{\tan u} = \lim_{u \rightarrow 0} \frac{u \cos u}{\sin u}$

13. $\lim_{x \rightarrow 0} \frac{\tan^{-1} x}{x} =$

- (a) 1
- (b) -1
- (c) 0
- (d) ∞

Handwritten solution for Q13:
 $u = \tan^{-1} x$
 $x = \tan u$
 $du = \frac{dx}{1+x^2}$
 $dx = (1+x^2) \cdot du$
 $= \lim_{x \rightarrow 0} \frac{(1+x^2)}{x} \cdot dx = \frac{1 + \tan^2 u}{\tan u} \cdot du$
 $= \int \frac{\sec^2 u}{\tan u} \cdot du$

14. $7^{1/2} =$

- (a) $\frac{7}{2}$
- (b) $\frac{7}{5}$
- (c) 5
- (d) 7

Handwritten solution for Q14:
 $= \int \frac{1}{\cos^2 u} \cdot \frac{\cos u}{\sin u} \cdot du = \int \frac{1}{\sin u \cos u} du$

15. The eccentricity of the conic section $\frac{x^2}{9} - \frac{y^2}{16} = 1$ is

- (a) $\frac{5}{3}$
- (b) $\frac{4}{3}$
- (c) $\frac{13}{15}$
- (d) $\frac{4}{3}$

Handwritten solution for Q15:
 $a = 3$
 $b = 4$
 $c^2 = a^2 + b^2 = 9 + 16 = 25$
 $c = 5$
 $e = \frac{c}{a} = \frac{5}{3}$

16. The length of the polar curve $r = 4 \cos \theta$, $0 \leq \theta \leq \frac{\pi}{2}$ is

- (a) π
- (b) 2π
- (c) 3π

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17. The surface area generated by revolving $r = 4 \cos \theta$ $0 \leq \theta \leq \frac{\pi}{2}$ about x-axis is

- (a) 2π
- (b) 4π
- (c) 8π
- (d) 16π

18. The Cartesian coordinates of the point $P(-4, \frac{\pi}{4})$ is

- (a) $(-2\sqrt{2}, 2\sqrt{2})$
- (b) $(-2\sqrt{2}, -2\sqrt{2})$
- (c) $(2\sqrt{2}, 2\sqrt{2})$
- (d) $(2\sqrt{2}, -2\sqrt{2})$

19. The angle θ that eliminate xy term in $2x^2 + \sqrt{3}xy + y^2 - 2y = 6$ is

- (a) $\frac{\pi}{3}$
- (b) $\frac{\pi}{4}$
- (c) $\frac{\pi}{6}$
- (d) $\frac{\pi}{2}$

20. $\sec^{-1}(-\sqrt{2}) = -\sqrt{2} = \sec y$

21. $\frac{\pi}{4} \quad -\sqrt{2} = \frac{1}{\cos y}$

22. $\frac{-\pi}{4}$

23. $\frac{\pi}{4}$

(24) $\frac{5\pi}{4}$

$\cos y = \frac{-1}{\sqrt{2}}$

$y = \frac{\pi}{4} + \frac{\pi}{4} = \frac{5\pi}{4}$

$\frac{\pi}{4} - \frac{\pi}{4} = -\frac{3\pi}{4}$

II (15 points): Evaluate the following integrals

$\int \sin^{\frac{5}{2}} x$

$\int \sec x$

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2016

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Birzeit University
Math. & Comp. Science Dept.
Math. 132

Dr. Hasan Yousef

Final Exam

Summer ~~2017~~

Student Name: _____

Number: _____

Section: _____

I (40points) : Circle the MOST correct answer:

1. If $\sinh x = \frac{-3}{4}$ then $\cosh x =$

- (a) $\frac{3}{4}$
- (b) $\frac{4}{5}$
- (c) $\frac{-3}{5}$
- (d) $\frac{-5}{4}$



2. The conic section with Foci $(\pm 1, 0)$ and vertices $(\pm 2, 0)$ is an

- (a) Ellipse
- (b) Parabola
- (c) Hyperbola
- (d) A circle

$c=1 \quad a=2$
 $a > c$
 $e < 1$

3. The conic section with eccentricity $\frac{1}{2}$ and directrix $x = 2$ has equation

- (a) $2x^2 + y^2 = 1$
- (b) $x^2 - y^2 = 1$
- (c) $y^2 - x^2 = 2$
- (d) $x^2 + \frac{4}{3}y^2 = 1$

$e = \frac{c}{a} = \frac{1}{2} \quad 2 \cdot \frac{1}{2} \quad \frac{a}{e} = 2$
 $\frac{a}{2} = \frac{c}{a} = \frac{1}{2}$
 $a^2 = 2c \quad a=1 \quad 2c=a$
 $\frac{c}{a} = \frac{1}{2}$
 $e = \frac{a}{2}$
 $a=1$

4. The directrix of the parabola $x = \frac{y^2}{2}$ is given by

- (a) $x = 1$
- (b) $y = \frac{1}{2}$
- (c) $y = \frac{1}{2}$
- (d) $x = \frac{1}{2}$

$\frac{x^2}{1} + \frac{4y^2}{3} = 1$
 $y^2 = 2x$
 $4px = 2x$
 $p = \frac{1}{2}$

$a = 2e$
 $= 2 \cdot \frac{1}{2}$
 $= 1$
 $e = ea = \frac{1}{2} \cdot 1$
 $e = \frac{1}{2}$

RAF

$D = \frac{a}{e} = 2$

$a = 2e$

$e = \frac{c}{a} = \frac{1}{2}$

$a = 1$

$a = 2c \rightarrow c = \frac{1}{2}$

$b^2 = a^2 - c^2$

$= 1 - \frac{1}{4}$
 $b = \frac{\sqrt{3}}{2}$

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2017



2016

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5. The conic section $x^2 + 4xy + \sqrt{2}y^2 + 5 = 0$ is

- (a) Ellips
- (b) Parabola
- (c) Hyperbola
- (d) A Circle

$$x^2 + \sqrt{2}y^2 = -4xy - 5$$

$$\frac{x^2 + \sqrt{2}y^2}{-4xy - 5} = 1$$

$$= \frac{x^2}{-4xy} + \frac{\sqrt{2}y^2}{-4xy} = \frac{5}{-4xy}$$

6. $x = \cos 2t, y = \sin^2 t, 0 \leq t \leq \frac{\pi}{4}$ represents

- (a) half of a circle
- (b) half of an Ellipse
- (c) a line segment
- (d) a parabola

$$-4x^2 - 4\sqrt{2}y^2 = -4xy$$

7. The slope of the Ellipse $x = 2 \sin t, y = 3 \cos t, 0 \leq t \leq 2\pi$ at the point $(\sqrt{2}, \frac{3}{\sqrt{2}})$ is

- (a) $-\frac{2}{3}$
- (b) 3
- (c) $\frac{3}{2}$
- (d) $\frac{3}{2}$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$4 \sin^2 t + 9 \cos^2 t = 1$$

$$\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2}{2\sqrt{2}}$$

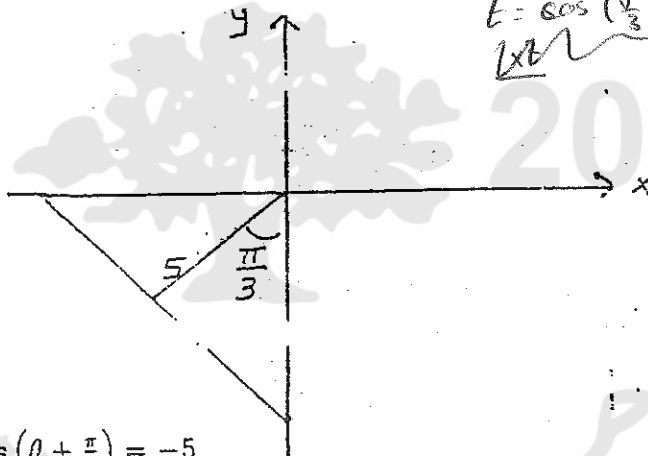
$$= \frac{1}{\sqrt{2}}$$

$$t = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$$

$$\frac{3}{3\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$t = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$$

8. An equation of the line in the figure is



- (a) $r \cos\left(\theta + \frac{\pi}{6}\right) = -5$
- (b) $r \sin\left(\theta + \frac{2\pi}{3}\right) = -5$
- (c) $r \cos\left(\theta + \frac{5\pi}{6}\right) = 5$
- (d) $r \sin\left(\theta + \frac{\pi}{6}\right) = -5$

9. The polar Equation of the circle with center $P(-2, \frac{\pi}{4})$ and radius 4 is

- (a) $r = -4 \sin\left(\theta + \frac{\pi}{4}\right)$
- (b) $r = -4 \sin\left(\theta + \frac{3\pi}{4}\right)$
- (c) $r = 4 \sin\left(\theta - \frac{3\pi}{4}\right)$
- (d) $r = 4 \sin\left(\theta - \frac{\pi}{2}\right)$

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2017



2016

مَجْلِسُ الطَّلَبَةِ

$$\int \frac{x^2 \cdot du}{2x\sqrt{1+x^2}} = \int \frac{x \cdot du}{2\sqrt{1+x^2}} = \int \frac{\sqrt{u} \cdot du}{2\sqrt{1+u}}$$

$$u = x^2 \\ du = 2x \cdot dx \\ dx = \frac{du}{2x}$$

Q2: (15 points) (a) $\int \frac{x^2 dx}{\sqrt{1+x^2}} =$

$$u = 1+x^2 \\ du = 2x \cdot dx \\ dx = \frac{du}{2x}$$

$$= \int \frac{x^2 \cdot du}{2x\sqrt{u}} = \int \frac{x \cdot du}{2\sqrt{u}} = \int \frac{\sqrt{u-1} \cdot du}{2\sqrt{u}}$$

$$= \frac{1}{2} \int \sqrt{\frac{u-1}{u}} \cdot du = \frac{1}{2} \int \sqrt{1 - \frac{1}{u}} \cdot du$$

$$= \int \frac{x^2 \cdot \sqrt{1+x^2} \cdot du}{x(u)} = \int \sqrt{u-1} \cdot du$$

$$= \int (u-1)^{\frac{1}{2}} \cdot du = \frac{2}{3} (u-1)^{\frac{3}{2}}$$

$$u = \sqrt{1+x^2} \\ du = \frac{2x \cdot dx}{2\sqrt{1+x^2}}$$

$$du = \frac{x \cdot dx}{\sqrt{1+x^2}}$$

$$dx = \frac{\sqrt{1+x^2} \cdot du}{x}$$

$$u = \sqrt{1+x^2}$$

$$u^2 = 1+x^2$$

$$x = \sqrt{u^2 - 1}$$

(b) $\int \frac{x^2+x+1}{x^3+x} dx =$

$$\frac{x^2+x+1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$$

(c) $\lim_{x \rightarrow \infty} \left(\frac{x-1}{x-2} \right)^x = \lim_{x \rightarrow \infty} \frac{(x-1)^x}{(x-2)^x} = \lim_{x \rightarrow \infty} \frac{(x-1)^{x+1}}{(x-2)^{x+1}}$

$$= \lim_{u \rightarrow \infty} \frac{(u)^{u+1}}{(u)^{u+2}}$$

$$= \lim_{u \rightarrow \infty} u^{(u+1)-(u+2)}$$

$$= \lim_{u \rightarrow \infty} u^{-1}$$

$$= \lim_{u \rightarrow \infty} \frac{1}{u} = 0$$

$$\frac{(x-1)^{x+1}}{(x-2)^{x+1}}$$

$$u = x-1$$

$$du = dx$$

$$x = u+1$$

$$x \rightarrow \infty \quad u = \infty$$

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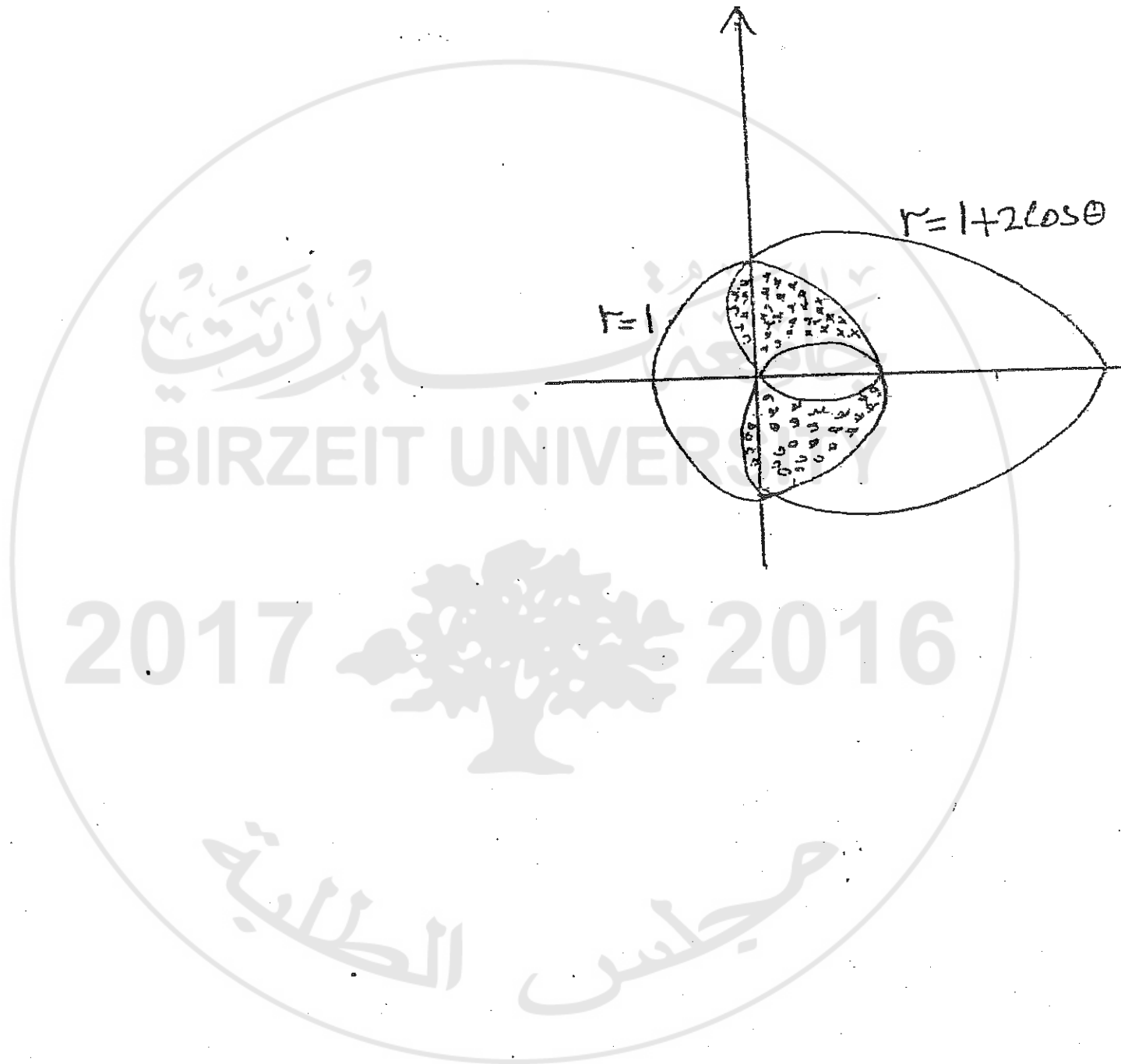
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IV (15 points): Find the area of the shaded region.



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2016

مَجْلِسُ الطَّلَبَةِ



Birzeit University
 Math. & Comp. Science Dept.
 Math. 132

Final Exam

Second Semester

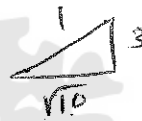
The Name of Discussion Teacher:

Student Name: _____ Number: _____ Section: _____

I (50 points) : Circle the MOST correct answer:

1. If $\sinh x = 3$ then $\cosh(-x) =$

- (a) $\sqrt{10}$
- (b) $-\sqrt{10}$
- (c) $\sqrt{8}$
- (d) $\sqrt{10}, -\sqrt{10}$



2. $\int \frac{ax+b}{x^3+x^2} dx$ is a rational function if

- (a) $b=0$
- (b) $a=b$
- (c) $a=-b$
- (d) $a=b$

3. If $2^{x^2} = 4 \cdot 2^x$ and $x > 0$ then $x =$

- (a) 2
- (b) -1
- (c) 2, -1
- (d) 4

$x \ln 2^x = x \ln 4 \cdot 2$

$x = \frac{\ln 4 \cdot 2}{\ln 2} = \ln 2 \cdot 2$

$x=1 \rightarrow u=e$
 $x=0 \rightarrow u=1$

4. $\int_0^1 \frac{e^x - e^{-x}}{e^x + e^{-x}} dx =$

- (a) $\tanh^{-1}(1)$
- (b) $\ln(e^2 + 1) - \ln 2$
- (c) $\ln(e^2 + 1) - \ln 2 - 1$
- (d) None of the above

$u = e^x$
 $du = e^x \cdot dx$

$= \int \frac{u - \frac{1}{u}}{u + \frac{1}{u}} \cdot \frac{du}{u} = \int \frac{u^2 - 1}{u(u^2 + 1)} \cdot \frac{du}{u}$

$= \int \frac{u^2 - 1}{u^2(u^2 + 1)} \cdot du = \int \frac{u^2 - 1}{u^2(u^2 + 1)} \cdot du$

$\frac{u^2 - 1}{u(u^2 + 1)} = \frac{A}{u} + \frac{B}{u^2 + 1}$

$u^2 - 1 = A(u^2 + 1) + B(u)$

$A = 1$

$u = 1$
 $2A + B = 0$
 $B = -2A$
 $B = -2$

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2017



2016

مَجْلَسُ الطَّلَبَةِ

$$= 2x \ln x - \frac{2(u e^u - e^u)}{\ln 3}$$

$$f(x) = u \oplus g(x) = e^u$$

$$f'(x) = 1 \oplus \int g(x) = e^u$$

$$\int \int g(x) = e^u$$

$$= \int \frac{\ln x^2}{\ln 3} \cdot dx = \frac{1}{\ln 3} \int \ln x^2 \cdot dx$$

$$u = \ln x^2$$

$$du = \frac{2x}{x^2} \cdot dx$$

$$5. \int \log_3 x^2 dx =$$

(a) $2x \log_3 x^2 - x + c$

(b) $2x \log_3 x - x + c$

(c) $\frac{2}{\ln 3} (x \ln x + x) + c$

(d) $\frac{2}{\ln 3} (x \ln x - x) + c$

$$= \frac{1}{\ln 3} \int u \cdot \frac{x}{2} \cdot dx$$

$$= \frac{1}{2 \ln 3} \int u \cdot e^u \cdot dx$$

$$z = e^u \quad dv = u \cdot du \quad x = \frac{e^u}{2}$$

$$dz = e^u \cdot du \quad u = \frac{u^2}{2} \quad \frac{u}{2} = \ln x$$

$$du = \frac{2}{x} \cdot dx$$

$$dx = \frac{x}{2} \cdot du$$

6. If $\int_0^b f(x) dx$ diverges and $\int_a^b g(x) dx$ diverges then $\int_a^b f(x) \cdot g(x) dx$

(a) converges always

(b) diverges always

(c) Can't decide

(d) Converges if $f(x) = g(x)$

$$= \frac{e^u \cdot u^2}{2} - \int \frac{u^2}{2} e^u \cdot du$$

7. A line $y = ax + b$ and a curve of a function $y = e^x$ can intersect in at most

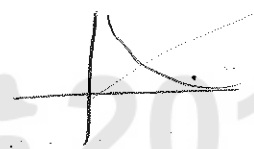
(a) 1 point

(b) 2 points

(c) 3 points

(d) 4 points

$$ax + b = e^x$$



8. If $\sin^{-1} x = \frac{\pi}{3}$ then $\cos^{-1} x =$

(a) $\frac{2\pi}{3}$

(b) $\frac{5\pi}{6}$

(c) $\frac{\pi}{3}$

(d) $\frac{\pi}{6}$

$$x = \frac{\sqrt{3}}{2}$$

$$x = \sin u$$

$$u = \sin^{-1} x$$

$$du = \frac{dx}{\sqrt{1-x^2}}$$

$$9. \int_0^{\frac{\pi}{2}} \sin x dx + \int_0^{\frac{\pi}{2}} \sin^{-1} x dx = \left[-\cos x \right]_0^{\frac{\pi}{2}} + \left[\sin u \right]_0^{\frac{\pi}{2}}$$

$$dx = \sqrt{1-x^2} \cdot du$$

$$= \cos u \cdot du$$

(a) π

(b) $\frac{\pi}{2}$

(c) 1

(d) $\frac{1}{2}$

$$= 0 + 1 + \frac{1}{\sqrt{2}} - 0$$

$$= \frac{2 + \sqrt{2}}{2}$$

$$\frac{x=1}{u=\pi/4} \mid \frac{x=0}{u=0}$$

10. One of the following is not an improper integral.

(a) $\int_0^{10} \frac{\sin x}{x} dx$

(b) $\int_{-\infty}^{\pi} x dx$

(c) $\int_0^1 \frac{1}{\sqrt{x}} dx$

(d) $\int_{-2}^{\infty} \frac{dx}{x^2}$

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11. $\int \sin^2 x \cos^3 x dx =$

(a) $\frac{\sin^3 x \cos^4 x}{12} + c$

(b) $\frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + c$

(c) $\frac{\cos^3 x \sin^4 x}{12} + c$

(d) $\frac{\sin^3 x}{15} (5 + 3 \sin^2 x) + c$

12. The function $f(x) = \ln[x + 2]$ has domain

(a) $(-2, \infty)$

(b) $(0, \infty)$

(c) $[-1, \infty)$

(d) $[0, \infty)$

13. $\int x^3 e^x dx =$

(a) $\frac{x^4}{4} e^x + c$

(b) $\frac{x^4}{4} e^x - e^x + c$

(c) $e^x(x^3 - 3x^2 + 6x - 6) + c$

(d) $e^x(-x^3 + 3x^2 - 6x + 6) + c$

14. If a particle moves on a parametric curve described by $x = t^2, y = \sqrt{1-t^2}$, $t \leq 1$, then

(a) The initial point is $(1, 0)$ and the end point is $(0, 1)$.

(b) The motion is clockwise.

(c) The motion is counter clockwise.

15. The total distance travelled by a particle moving on the curve $r = 2 \sin \theta$ from $\theta = 0$ to $\theta = 2\pi$ is

(a) 4π

(b) 2π

(c) π

(d) 8π

16. The slope of the tangent lines to the curve $r = 1 - 2 \cos \theta$ at the origin are

(a) 0

(b) $\pm\sqrt{3}$

(c) ± 1

(d) not defined

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$$y = \pm \frac{b}{a}x$$

$$a = 4 \quad c = 5 \\ b = 3$$

$$e = \frac{c}{a} = \frac{5}{4}$$

$$x = \pm \frac{a}{e}$$

17. The directrices of the hyperbola $\frac{(x-4)^2}{16} - \frac{(y-1)^2}{9} = 1$ are

- (a) $y = \pm \frac{16}{5}$
- (b) $x = \pm \frac{16}{5}$
- (c) $x = \frac{-26}{5}, x = \frac{10}{5}$
- (d) $y = \frac{-26}{5}, y = \frac{10}{5}$

$$D = \frac{a}{e} = \frac{4}{\frac{5}{4}} = \frac{16}{5}$$

$$\frac{4}{\frac{5}{4}}$$

18. The polar curves $r = \cos 2\theta, r = \frac{1}{2}$ intersects in

- (a) 1 point
- (b) 2 points
- (c) 4 points
- (d) 8 points

$$x = \pm \frac{16}{5} \\ x = +\frac{16}{5} - 2 \quad x = -\frac{16}{5} - 2$$

19. The circles $x^2 + y^2 = 1$ and $(x-1)^2 + y = a^2$ intersect in two point if:

- (a) $a = 2$
- (b) $2 < a$
- (c) $a < 2$
- (d) $0 < a < 2$

$$x \rightarrow 0 \\ u \rightarrow \infty$$

20. $\lim_{x \rightarrow 0^+} \frac{\ln x}{\sqrt{x}} =$

- (a) 0
- (b) $-\infty$
- (c) ∞
- (d) 1

$$u = \ln x \\ du = \frac{dx}{x}$$

$$dx = x \ln x \\ x = e^u \\ u = \sqrt{x}$$

$$= \frac{1}{x} = \frac{2\sqrt{x}}{x} = \infty$$

II (15 points): 1. $\lim_{x \rightarrow \infty} \left(\frac{x-1}{x-2}\right)^x$

$$= \lim_{x \rightarrow \infty} \frac{(x-1)^x}{(x-2)^x} = \frac{x \ln(x-1)}{x \ln(x-2)} = \frac{x - \frac{x^2}{2} + \frac{x^3}{3} \dots}{\ln 2 - \frac{x}{2} - \frac{x^2}{4} \dots}$$

$$f(x) = \ln(x-2) \quad f(e) = \ln 2 \\ f'(x) = \frac{1}{x-2} \quad f'(e) = \frac{1}{e-2} = \ln 2 \\ f''(x) = \frac{-1}{(x-2)^2} \quad f''(e) = \frac{-1}{e-2}$$



2. Test for convergence $\int_{-\infty}^{\infty} \frac{dx}{e^x + e^{-x}}$.

$$= \int_{-\infty}^{\infty} \frac{dy}{u(u+\frac{1}{u})} = \int_{-\infty}^{\infty} \frac{du}{u(u^2+1)}$$

$$= \int_{-\infty}^{\infty} \left[\tan^{-1} u \right]_{-\infty}^0 + \left[\tan^{-1} \frac{1}{u} \right]_0^{\infty}$$

$$= \left[\tan^{-1}(e^x) \right]_{-\infty}^0 + \left[\tan^{-1}(e^{-x}) \right]_0^{\infty}$$

$$= \frac{\pi}{4} - \left(-\frac{\pi}{4}\right) + \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{2}$$

$$u = e^x$$

$$du = e^x dx$$

$$dx = \frac{du}{e^x} = \frac{du}{u}$$

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3. Evaluate $\int \frac{\sqrt{9-x^2}}{x} dx$.

~~$$\int \frac{\sqrt{u}}{2x^2} du$$~~

~~$$\int \frac{\sqrt{u}}{2(4u)} du$$~~

~~$$\int \frac{\sqrt{9-u}}{2u} du$$~~

~~$$u = 9/x^2$$~~
~~$$du = -2x dx$$~~
~~$$dx = \frac{du}{-2x}$$~~

$$u = x^2$$

$$du = 2x dx$$

$$dx = \frac{du}{2x}$$

~~$$a^2 = x^2$$~~

$$x = a \sin \theta$$

$$a^2 - x^2 = a^2 - a^2 \sin^2 \theta = a^2 \cos^2 \theta$$

$$dx = a \cos \theta d\theta$$

$$= \int \frac{\sqrt{a^2 - a^2 \sin^2 \theta}}{a \sin \theta} a \cos \theta d\theta$$

$$= \int \frac{\cos^2 \theta d\theta}{\sin \theta} = \int \cot \theta \csc \theta d\theta = \csc \theta + c$$

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III (20 points): (a) Sketch the graph of $r = \frac{8}{2 - \sin \theta}$ and indicate the center, vertices, foci and directrices.

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(b) Use part (a) to sketch the graph of $r = \frac{8}{2 - \sin(\theta + \frac{\pi}{4})}$

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IV (20 points): (a) Sketch the graph of the conic section $r = \frac{8}{2 - 4 \cos \theta}$ indicating vertices foci and directrices in polar coordinates.

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(b) Use (a) to sketch the graph of $r = \frac{8}{2 - 4 \cos(\theta + \frac{\pi}{4})}$ indicating vertices, foci and directrices in polar coordinates.

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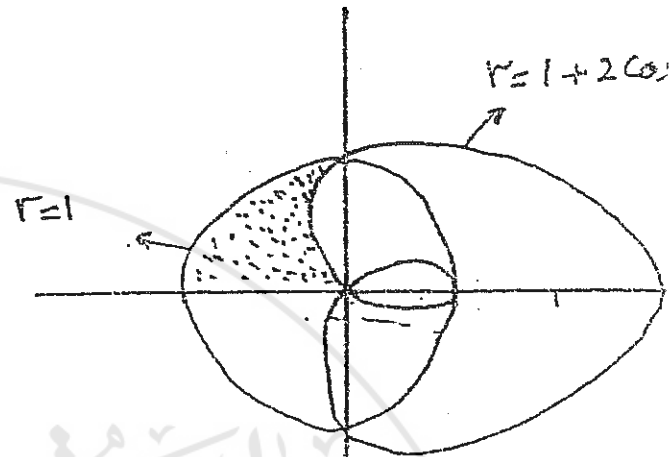
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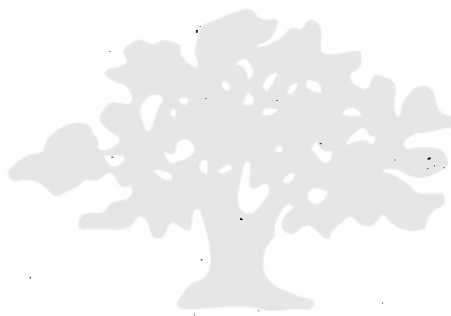
V (15 points): Find the shaded area.



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IV (20 points): (a) Sketch the graph of the conic section $r = \frac{8}{2 - 4 \cos \theta}$ indicating vertices foci and directrices in polar coordinates.

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Birzeit University- Mathematics Department
Math 132

Dr. Marwan Alogeili (Sec.2) and Dr. Marwan Awartani (Sec.1&3)

Fall 2002/2003

Third Exam

Name: Ali Nusef Tagatga

Number: 1011273

There are 10 (T/F) questions and 14 multiple choice. Calculators are not allowed.

Question 1 Circle the most correct answer:

$\lim_{n \rightarrow \infty} \frac{1}{n+10^6}$

$\frac{1}{n} = 1/n^1$

70

1. $\sum_{n=1}^{\infty} \frac{1}{n+(10)^n}$

- (a) Converges to 0.
- (b) Converges by nth term test.
- (c) Diverges.
- (d) None.

2. One of the following p-series converges:

- (a) $\sum_{n=1}^{\infty} \frac{1}{n^{1.1}}$
- (b) $\sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$
- (c) $\sum_{n=1}^{\infty} \frac{1}{n}$
- (d) $\sum_{n=1}^{\infty} \frac{1}{n^{3/4}}$

$p > 1$

3. The sequence $a_n = (1 - \frac{1}{n^2})^n$

- (a) Diverges.
- (b) Converges to -1.
- (c) Converges to 1.
- (d) None of the above.

$\lim_{n \rightarrow \infty} (1 - \frac{1}{n^2})^n$
 $n \ln(1 - \frac{1}{n^2}) = \ln(1 - \frac{1}{n^2}) + \frac{1}{n^2}$
 $= 0 + 0 = 0$
 $e^0 = 1$

4. $\sum_{n=1}^{\infty} \tan^{-1}(n+1) - \tan^{-1}n =$

- (a) $\pi/2$.
- (b) $\pi/4$.
- (c) $-\pi/4$.
- (d) None of the above.

$\tan^{-1}(2) - \tan^{-1}(1) + \tan^{-1}(3) - \tan^{-1}(2)$

5. $\sum_{n=2}^{\infty} \frac{\ln n}{n}$

- (a) Diverges by integral test.
- (b) Diverging by directly comparing it with $\sum \frac{1}{n}$.
- (c) Diverging by limit comparison test with $\sum \frac{1}{n}$.
- (d) All of the above.

$\frac{1}{2} \ln^2 b - \frac{1}{2} \ln^2 2$

$\frac{\ln n}{n} > \frac{1}{n}$
 $\lim_{n \rightarrow \infty} \frac{\ln n}{n} = 0$

$\int \frac{\ln u}{u} = \int \frac{u}{u} x du$
 $= \frac{1}{2} u^2$

Handwritten notes and calculations:

- $\lim_{n \rightarrow \infty} (1 - \frac{1}{n^2})^n$
- $\ln f(x) = \lim_{n \rightarrow \infty} \ln(1 - \frac{1}{n^2})$
- $\frac{1}{1 - 1/n^2} \cdot 0 \cdot \frac{1}{(n^2)}$
- $\lim_{n \rightarrow \infty} \frac{-1/n}{1 - 1/n^2} = \lim_{n \rightarrow \infty} \frac{-1/n}{1} = 0$
- $\lim_{n \rightarrow \infty} \ln f(x) = 0$
- $\Rightarrow \ln f(x) = 0$
- $\Rightarrow \lim_{n \rightarrow \infty} f(x) = e^0 = 1$
- $(\tan^{-1} 2 - \tan^{-1} 1) + (\tan^{-1} 3 - \tan^{-1} 2) + \dots + (\tan^{-1} n - \tan^{-1} (n-1))$
- $S_n = -\tan^{-1} 1 + \tan^{-1}(n+1) = \frac{\pi}{4}$

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$$\frac{1 + \cos^2 n}{n^2} < \frac{2}{n^2}$$

6. $\sum_{n=1}^{\infty} \frac{1 + (\cos n)^2}{n^2}$

- (a) Converges.
- (b) Diverges.
- (c) Diverges by the nth term test.
- (d) Cannot determine.

7. $\sum_{n=1}^{\infty} e^{-n} = \frac{1}{e} + \frac{1}{e^2} + \frac{1}{e^3} + \dots$

- (a) $\frac{e}{e-1}$
- (b) $\frac{1}{e-1}$
- (c) $\frac{1}{1-e}$
- (d) None of the above.

$$\left(\frac{1}{e}\right)^n = \frac{1}{e^n} = \frac{1}{e \cdot e^{n-1}} = \frac{1}{e} \cdot \frac{1}{e^{n-1}}$$

$$= \frac{1}{1 - \frac{1}{e}} = \frac{1}{\frac{e-1}{e}} = \frac{e}{e-1}$$

8. One of the following is true:

- (a) $\sum_{n=1}^{\infty} \frac{3^n}{n^{3/2}}$ diverges by ratio test.
- (b) $\sum_{n=1}^{\infty} \frac{1}{n+1}$ converges by nth root test. \times
- (c) $\sum_{n=1}^{\infty} 2^n$ converges. \times
- (d) $\sum \frac{1}{\ln n}$ is a geometric series. \times

$$\lim_{n \rightarrow \infty} \frac{3^{n+1}}{(n+1)^{3/2+1}} \cdot \frac{n^3 2^n}{2^n}$$

$$= \lim_{n \rightarrow \infty} \frac{3^{n+1}}{(n+1)^3} = \frac{3}{2}$$

9. The nth partial sum of the series $\sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}}\right)$ is

- (a) $s_n = 1 - \frac{1}{\sqrt{2}}$
- (b) $s_n = \frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}}$
- (c) $s_n = 1 - \frac{1}{\sqrt{n+1}}$
- (d) None of the above.

$$\frac{1}{\sqrt{1}} - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}} + \dots + \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}}\right)$$

10. If $a_1 = 2, a_{n+1} = \frac{2}{3}a_n$ then $\sum_{n=2}^{\infty} a_n$

- (a) Converges by ratio test.
- (b) Diverges.
- (c) Converges by integral test.
- (d) None of the above.

$$2 + 2(2) + 1(4) + \dots$$

$$\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \frac{\frac{2}{n} a_n}{a_n} = \frac{2}{n} = 0$$

11. The series $\sum_{n=0}^{\infty} (\ln x)^n$ converges if

- (a) $-1 < x < 1$
- (b) $-e < x < e$
- (c) $e^{-1} < x < e$
- (d) None.

$$\sum_{n=0}^{\infty} (\ln x)^n$$

$$a_1 = (\ln x)^0 = 1$$

$$a_2 = (\ln x)^1$$

$$a_3 = (\ln x)^2 \Rightarrow r = (\ln x)$$

$$s_n = \frac{1}{1 - (\ln x)}$$

$$e^{-1} < \ln x < 1$$

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12. One of the following series converges:

- (a) $\sum_{n=1}^{\infty} n^2$
- (b) $\sum_{n=1}^{\infty} \frac{n+1}{n}$
- (c) $\sum_{n=1}^{\infty} \frac{1}{2^n}$
- (d) $\sum_{n=1}^{\infty} \ln(1/n)$

- B E

13. $\sum_{n=1}^{\infty} \frac{-5}{n}$

- (a) Converges to 0.
- (b) Converges to 1.
- (c) Diverges.
- (d) None of the above.

14. $\sum_{n=1}^{\infty} \frac{2^n}{n^2}$

- (a) Converges.
- (b) Diverges.
- (c) Converges to 0.
- (d) None.

$$\lim \frac{2^{n+1}}{(n+1)^2} = \lim \frac{2 \cdot 2^n}{n^2 + 2n + 1}$$

$$= 2 \lim \frac{2^n}{n^2 + 2n + 1}$$

$= 2 > 1 \Rightarrow \text{div.}$

$$\frac{(-1)^{n+2}}{(-1)^{n+1}}$$

$$= (-1)^{n+2-n-1}$$

$$= (-1)^1$$

Question 2. Answer by True or False:

1. The series $\sum_{n=1}^{\infty} (-1)^{n+1}$ diverges.
2. If $\lim_{n \rightarrow \infty} (a_n)^{1/n} = 1$ then $\sum_{n=1}^{\infty} a_n$ converges.
3. Any increasing sequence and bounded from above converges.
4. $\lim_{n \rightarrow \infty} \frac{2^n}{n!} = 0$.
5. The sequence $a_n = 1 + (-1)^n$ diverges.
6. If $\lim_{n \rightarrow \infty} a_n = 0$ then $\sum_{n=1}^{\infty} a_n$ converges.
7. The series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is called the harmonic series.
8. The sequence $a_n = \frac{3n+1}{n+1}$ is nondecreasing.
9. The series $\sum_{n=1}^{\infty} (\sin x)^n$ converges for any value of x .
10. If $\sum_{n=1}^{\infty} a_n$ converges then $\sum_{n=1}^{\infty} k a_n$, k is any constant, converges.

- (True) ✓ $\sum_{n=2}^{\infty} (-1)^n$
- (False) ✓ $(-1)^n$
- (True) ✓ $(-1)^n$
- (True) ✓ $(-1)^n$
- (True) ✓ $(-1)^n$
- (False) ✓ $(-1)^n$
- (False) ✓ $(-1)^n$
- (True) ✓ $(-1)^n$
- (False) ✓ $(-1)^n$
- (True) ✓ $(-1)^n$

30

$$\lim_{n \rightarrow \infty} \frac{(-1)^{n+2}}{(-1)^{n+1}} = -1 \text{ diverges}$$

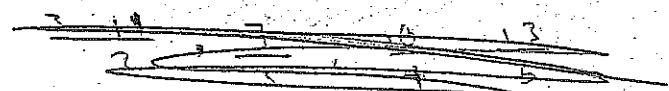
0, 1, 0, 1

$$\sum_{n=2}^{\infty} (-1)^n$$

$$\frac{3(n+1) - (3n)}{(n+1)^2}$$

$$\frac{3n+3-3n}{(n+1)^2}$$

$$= \frac{3}{(n+1)^2}$$



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Birzeit University
Math. & Comp. Science Dept.
Math: 132

Third Hour Exam

Second Semester ~~XXXXXX~~

The Name of Discussion Teacher: _____

Student Name: _____ Number: _____ Section: _____

I (40 points): Circle the MOST correct answer:

1. The directrix of the parabola $y = 2x^2 + 4x + 1$ is
 - (a) $y = -\frac{1}{8}$
 - (b) $y = \frac{1}{8}$
 - (c) $y = -\frac{9}{8}$
 - (d) $y = \frac{9}{8}$

2. A circle of radius a and an ellipse of major semiaxis a both centered at the origin meets in
 - (a) 1 points
 - (b) 1 points
 - (c) 2 point
 - (d) 0 point

3. The quadratic equation $x^2 + 2xy + y^2 - 2x + 2y + 10 = 0$ represents
 - (a) Parabola
 - (b) Ellipse
 - (c) Hyperbola
 - (d) Circle

4. A parametrization of the line segment with initial point $(0, 1)$ and terminal point $(1, 0)$ is
 - (a) $x = \sin^2 t, y = \cos^2 t, 0 \leq t \leq \frac{\pi}{2}$
 - (b) $x = 1 - t, y = t, 0 \leq t \leq 1$
 - (c) $x = t, y = 1 + t, -1 \leq t \leq 0$
 - (d) $x = 1 + t, y = t, 0 \leq t \leq 1$

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5. The Parametrization $x(t) = -\cos t$, $y(t) = \sin t$, $0 \leq t \leq 2\pi$

- (a) A circle with initial point $(1,0)$ counterclockwise.
- (b) A circle with initial point $(-1,0)$ counterclockwise.
- (c) A circle with initial point $(-1,0)$ clockwise.
- (d) A circle with initial point $(1,0)$ clockwise.

6. The length of the curve $x(t) = \sin t$, $y(t) = 1 + \cos t$, $0 \leq t \leq \frac{\pi}{2}$ is:

- (a) π
- (b) 2π
- (c) $\frac{\pi}{4}$
- (d) $\frac{\pi}{2}$

7. The slope of the tangent line to the curve $y(t) = t^2 - \sin t$, $x(t) = \cos t$, at $t = \frac{\pi}{2}$ is:

- (a) $\frac{\pi^2}{2}$
- (b) $\frac{-1}{\pi}$
- (c) -2
- (d) $-\pi$

8. Which one of the following points lies on the curve $r = \cos 2\theta$

- (a) $(0,0)$
- (b) $(1, \frac{\pi}{2})$
- (c) $(\frac{1}{2}, \frac{\pi}{3})$
- (d) all

9. The graph of $r = 2 \csc \theta$ is

- (a) Circle
- (b) Ellipse
- (c) Hyperbola
- (d) Parabola

10. The curves $\theta = \frac{\pi}{2}$ and $r = \theta$

- (a) never meets.
- (b) intersect in one point.
- (c) intersect in infinitely many points.
- (d) are identical.

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11. $7^{\log_7 5} =$

- (a) $\frac{5}{7}$
- (b) $\frac{7}{5}$
- (c) 5
- (d) 7

12. The directrix of the parabola $y = 2x^2 + 4x + 1$ is

- (a) $y = -\frac{1}{8}$
- (b) $y = \frac{7}{8}$
- (c) $y = -\frac{9}{8}$
- (d) $y = \frac{9}{8}$

13. The quadratic equation $x^2 + 2xy + y^2 - 2x + 2y + 10 = 0$ represents

- (a) Parabola
- (b) Ellipse
- (c) Hyperbola
- (d) Circle

14. A parametrization of the line segment with initial point (0,1) and terminal point (1,0) is

- (a) $x = \sin^2 t$, $y = \cos^2 t$, $0 \leq t \leq \frac{\pi}{2}$
- (b) $x = 1 - t$, $y = t$, $0 \leq t \leq 1$
- (c) $x = 1$, $y = 1 + t$, $-1 \leq t \leq 0$
- (d) $x = 1 + t$, $y = 1$, $0 \leq t \leq 1$

15. $\frac{\log_2(x)}{\log_8(x)} =$

- (a) $\frac{2}{8}$
- (b) $\frac{1}{3}$
- (c) \log_4^2
- (d) 3

16. $\sec^{-1}(-2)$

- (a) $\frac{2\pi}{3}$
- (b) $-\frac{\pi}{3}$
- (c) $-\frac{2\pi}{3}$
- (d) $\frac{3\pi}{4}$

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17. $\lim_{x \rightarrow 0^+} (\ln x - \ln(\sin x))$

- (a) 0
- (b) -1
- (c) +1
- (d) Doesn't exist.

18. $\lim_{x \rightarrow \infty} (\ln x)^{\frac{1}{x}}$

- (a) ∞
- (b) 0
- (c) 1
- (d) Doesn't exist

19. $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\sec x}{\tan x} =$

- (a) 0
- (b) 1
- (c) ∞
- (d) Doesn't exist

20. The order of the functions $x^2, e^x, \ln x$ from slowest growing to fastest growing as $x \rightarrow 0$ is

- (a) $x^2, \ln x, e^x$
- (b) $e^x, x^2, \ln x$
- (c) $x^2, e^x, \ln x$
- (d) $\ln x, x^2, e^x$

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II (20 points): Suppose the path of a moving particle in a plane is described by

$$\begin{aligned}x(t) &= 3 + 4 \sin(t) \\y(t) &= 2 + 5 \cos(t), \quad 0 \leq t \leq \pi\end{aligned}$$

1. Sketch the path of motion and determine the direction.
2. Find the equation of the tangent line at $t = \frac{\pi}{3}$.

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Question #4:

Graph the conic section $2x^2 + 3xy + 2y^2 - 1 = 0$ in the xy -plane indicating the center, the vertices and the foci in the xy coordinates.

$$2x^2 + 3xy + 2y^2 - 1 = 0$$

$$B^2 - 4AC = 9 - 4(2)(2) < 0 \text{ ellips}$$

$$\cot 2\alpha = \frac{A-C}{B} = \frac{2-2}{3} = 0$$

$$2\alpha = \frac{\pi}{2}$$

$$\alpha = \frac{\pi}{4}$$

$$x = x' \cos \frac{\pi}{4} - y' \sin \frac{\pi}{4}$$

$$y = \frac{x'}{\sqrt{2}} + \frac{y'}{\sqrt{2}}$$

$$y = x' \sin \frac{\pi}{4} + y' \cos \frac{\pi}{4}$$

$$= \frac{x'}{\sqrt{2}} + \frac{y'}{\sqrt{2}}$$

Equation became

$$2\left(\frac{x'-y'}{\sqrt{2}}\right)^2 + 3\left(\frac{x'}{\sqrt{2}} - \frac{y'}{\sqrt{2}}\right)\left(\frac{x'+y'}{\sqrt{2}}\right) + 2\left(\frac{x'+y'}{\sqrt{2}}\right)^2 - 1 = 0$$

$$2\left(\frac{x'^2 + y'^2 - 2xy'}{2}\right) + 3\left(\frac{x'^2 - y'^2}{2}\right) + 2\left(\frac{x'^2 + y'^2 + 2xy'}{2}\right) - 1 = 0$$

$$x'^2 + y'^2 + \frac{3x'^2}{2} - \frac{3y'^2}{2} + x'^2 + y'^2 = 1$$

$$\frac{7}{2}x'^2 + \frac{1}{2}y'^2 = 1$$

$$\frac{x'^2}{\frac{2}{7}} + \frac{y'^2}{2} = 1$$

~~$$\frac{x'^2}{\frac{2}{7}} + \frac{y'^2}{2} = 1$$~~

Foci

~~$$c = \sqrt{\frac{2}{7} - 2} = \sqrt{-\frac{10}{7}}$$~~

$$c = \sqrt{\frac{14}{49} - \frac{4}{49}} = \sqrt{\frac{10}{49}} = \frac{\sqrt{10}}{7} \approx 0.45$$

$$c = \sqrt{a^2 - b^2}$$

$$= \sqrt{2 - \frac{2}{7}} = \sqrt{1.714} = 1.3$$

Foci at $x'y'$ plan = $(0, \pm 1.3)$

at xy plan

$$y = x' \sin \frac{\pi}{4} + y' \cos \frac{\pi}{4}$$

$$= 0 + 1.3 \cos \frac{\pi}{4}$$

$$= \frac{1.3}{\sqrt{2}}$$

$$x = x' \cos \frac{\pi}{4} - y' \sin \frac{\pi}{4}$$

2

Foci in xy plan = $(\pm \frac{1.3}{\sqrt{2}}, \pm \frac{1.3}{\sqrt{2}})$

Foci

$$\left(\frac{1.3}{\sqrt{2}}, \frac{1.3}{\sqrt{2}}\right)$$

$$\left(\frac{1.3}{\sqrt{2}}, -\frac{1.3}{\sqrt{2}}\right)$$

$$\left(-\frac{1.3}{\sqrt{2}}, \frac{1.3}{\sqrt{2}}\right)$$

$$\left(-\frac{1.3}{\sqrt{2}}, -\frac{1.3}{\sqrt{2}}\right)$$



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~~vertices~~

vertices

$$a = (0, \pm\sqrt{2})$$

$$x = x' \cos \alpha - y' \sin \alpha$$

$$x = 0 - \frac{\sqrt{2}}{\sqrt{2}} = -1$$

$$y = x' \sin \alpha + y' \cos \alpha$$

$$= \frac{\sqrt{2}}{\sqrt{2}} = 1$$

3 Vertices in xy plane

~~$a_1 = (1, 1)$~~

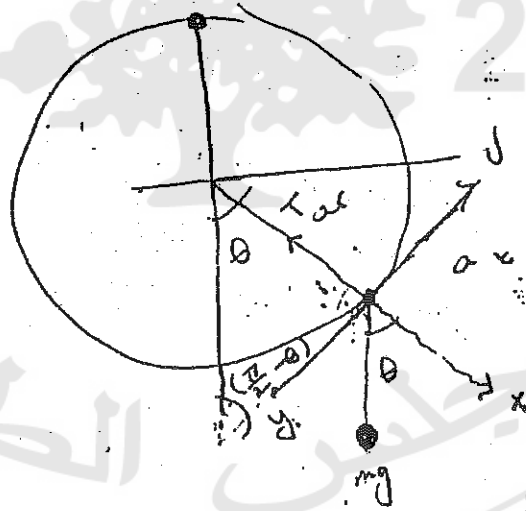
$$a_1 = (-1, 1)$$

$$a_2 = (1, -1)$$

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$$\frac{mv^2}{R} = \frac{T}{\cos \theta}$$
$$v = \sqrt{Rg \cos \theta}$$



$$a = -mg \sin \theta$$

$$mg \cos \theta + \frac{mv^2}{R} = T \quad (1)$$

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Q#7:

Find the series radius of convergence . For what values of x does the series converge :

a) Conditionally

b) absolutely

(i) $\frac{(x-2)^n}{n}$

(ii) $\frac{(-1)^n x^n}{n!}$

Q#8:

a) Find the maclurin series for $f(x) = \ln(1+x^2)$.

b) How many terms are supposed to be used to get an estimate $f(0.2)$ with error less than 10^{-6} .

#9:

Evaluate: $\sum_{n=1}^{\infty} nx^n$ if $|x| < 1$

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MATH DEPARTMENT

MATH 132 TEST THREE

TIME: 60 Min.

JANUARY 2008

NAME: George Hannouch

NUMBER: 106 15 15

SECTION: 1

Instructor's name: Dr. Rimon Tad'oh

82

QUESTION ONE: (MULTIPLE CHOICE) [40 POINTS]

CIRCLE THE RIGHT ANSWER:

28
30
10
14

1. Let $f(x) = \sum_{n=0}^{\infty} x^n$. The interval of convergence of the definite integral 0 to x,

$\int_0^x f(t) dt$ is

- (A) $x = 0$ only
- (B) $|x| \leq 1$
- (C) $-\infty < x < \infty$
- (D) $-1 \leq x < 1$
- (E) $-1 < x < 1$

$$\frac{x^{n+1}}{n+1} = \frac{x^{n+1}}{n+1} - \frac{x^n}{n+1} + \frac{x^n}{n+1}$$

$$\frac{x^{n+1}}{n+1} - \frac{x^n}{n+1}$$

$\ln |1+x|$

2. The coefficient of x^4 in the Maclaurin series for $f(x) = e^{-x/2}$ is

- (A) $-1/24$
- (B) $1/24$
- (C) $1/96$
- (D) $-1/384$
- (E) $1/384$

$$\frac{(-1)^n}{n!} \left(\frac{x}{2}\right)^n$$

$$\frac{1}{32} \frac{1}{4 \times 3 \times 2} x^4$$

$$\frac{1}{e^{x/2}}$$

$$\frac{1}{2 \cdot 2^2}$$

$$\frac{1}{4 \cdot 2^2}$$

3. Which of the following series diverges?

- (A) $\sum 1/n^2$
- (B) $\sum 1/(n^2 + n)$
- (C) $\sum n/(n^3 + 1)$
- (D) $\sum \frac{n}{\sqrt{4n^2 - 1}}$
- (E) none of the preceding.

conv
conv.

$$\frac{x}{\sqrt{4x^2 - 1}}$$

$$\frac{1}{4}$$

$$-\frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \dots$$

5

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4. For which of the following series does the Ratio Test fail?

(A) $\sum 1/n!$

(B) $\sum n/2^n$

(C) $1 + 1/2^{3/2} + 1/3^{3/2} + 1/4^{3/2} + \dots$

(D) $(\ln 2)/2^2 + (\ln 3)/2^3 + (\ln 4)/2^4 + \dots$

(E) $\sum n^n/n!$

$$\left(\frac{1}{2}\right)^{3/2}$$

$$\frac{1}{3^{3/2}} \times 2^{3/2}$$

$$\frac{\ln 3}{2^3} \frac{2^3}{\ln 2}$$

$$\frac{1}{2} \frac{\ln 4}{\ln 3}$$

$$\left(\frac{2}{3}\right)^{3/2}$$

$$\left(\frac{3}{4}\right)^{3/2}$$

5. Which of the following alternating series diverges?

(A) $\sum (-1)^{n-1}/n$

(B) $\sum (-1)^{n+1}(n-1)/(n+1)$

(C) $\sum (-1)^{n+1}/\ln(n+1)$

(D) $\sum \frac{(-1)^{n-1}}{\sqrt{n}}$

(E) $\sum (-1)^{n-1}n/n^2 + 1$

$$2\sqrt{2n}$$

6. Which of the following series converges conditionally?

(A) $3 - 1 + 1/9 - 1/27 + \dots$

(B) $\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} - \dots$

(C) $1/2^2 - 1/3^2 + 1/4^2 - \dots$

(D) $1 - 1.1 + 1.21 - 1.332 + \dots$

(E) $1/(1*2) - 1/(2*3) + 1/(3*4) - 1/(4*5) + \dots$

$$\frac{1}{3}$$

$$\frac{1}{9}$$

$$\frac{1}{3}$$

7. Suppose $f(x)$ is a function with Taylor series converging to $f(x)$ for all $x \in \mathbb{R}$.

If $f(0) = 2$, $f'(0) = 2$ and $f''(0) = 3$ for $n \geq 2$ then $f(x) =$

(A) $3e^x + 2x - 1$

(B) $e^{3x} + 2x + 1$

(C) $e^{3x} - x + 1$

(D) $3e^x - x - 1$

(E) $3e^x + 5x + 5$

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$$\frac{\ln(\frac{1}{n^3})}{\frac{1}{n^3}}$$

$$\frac{\frac{1}{n^3}}{\frac{1}{n^3}}$$

$$\frac{1}{n^3}$$

8. Which of the following series converge?

- (I) $\sum_{n=1}^{\infty} \frac{\ln(n^{-3})}{n^{-3}}$ (II) $\sum_{n=1}^{\infty} \frac{\ln 3}{3n}$ (III) $\sum_{n=1}^{\infty} \frac{n}{3^n}$

- (A) I only (B) II only (C) III only
 (D) None (E) I and III

9. What is the Taylor series for $f(x) = e^x$ about $x = 1$?

- (A) $\sum_{n=0}^{\infty} \frac{-(x-1)^n}{n!}$ (B) $\sum_{n=0}^{\infty} \frac{-e(x-1)^n}{n!}$ (C) $\sum_{n=0}^{\infty} \frac{(x-1)^n}{n!e}$

- (D) $\sum_{n=0}^{\infty} \frac{e(x-1)^n}{n!}$ (E) $\sum_{n=0}^{\infty} \frac{(x-1)^n}{n!}$

10. Let $\{a_n\}$ be a sequence of positive real numbers such that

$$\frac{1}{2} \leq \frac{a_{n+1}}{a_n} \leq \frac{n+4}{2n+1} \text{ for all } n. \text{ Then } \lim_{n \rightarrow \infty} a_n =$$

- (A) 0 (B) 1/2 (C) 1
 (D) 2 (E) 4

$$\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} \leq \frac{1}{2}$$

$\Rightarrow \sum a_n$ converges

$$\Rightarrow \lim_{n \rightarrow \infty} a_n = 0$$

QUESTION TWO: [30 points]

Test the following series for convergence or divergence. State the test you are using, and verify that it applies. Determine whether the convergence is absolute or conditional.

(a) $\sum_{n=3}^{\infty} \frac{1}{n(\ln n)^{100}}$ \Rightarrow Converges by Integral test

$$\int_3^{\infty} \frac{1}{n(\ln n)^{100}} = \frac{1}{99(\ln n)^{99}} \Big|_3^{\infty}$$

$$= \frac{1}{99(\ln n)^{99}} \Big|_3^{\infty} = \frac{1}{99(\ln 3)^{99}} \Rightarrow \text{Integral Converges}$$

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(b) $\sum_{n=0}^{\infty} \frac{n^{2n}}{(1+2n^2)^n} \Rightarrow$ Converges by the n th Root test absol.

$$\lim_{n \rightarrow \infty} \sqrt[n]{\frac{n^{2n}}{(1+2n^2)^n}}$$

$$= \lim_{n \rightarrow \infty} \frac{n^2}{1+2n^2} = \boxed{\frac{1}{2}} < 1 \Rightarrow \text{Converges}$$

(c) $\sum_{n=1}^{\infty} \frac{n^3 - \sqrt{n+6}}{n^4 - n - 3}$ diverges by limit comparison test

$$\lim_{n \rightarrow \infty} \frac{n^3 - \sqrt{n+6}}{n^4 - n - 3} \sim \frac{1}{n}$$

$$\lim_{n \rightarrow \infty} \frac{n^4 - n^{\frac{3}{2}} + 6n}{n^4 - n - 3}$$

$$= 1 \Rightarrow \text{both diverge or converge}$$

$\frac{1}{n}$ diverges. (power series with $p=1$)

\Rightarrow both diverge

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QUESTION THREE: [14 points]

Consider the power series $\sum_{n=1}^{\infty} \frac{(-2)^n}{\sqrt{n}} (x+3)^n$

- (a) When $x = -4$ does this series converge or diverge?
 (b) Determine all values for which the series converges.

$$\lim_{n \rightarrow \infty} \sqrt[n]{\frac{(-2)^n}{\sqrt{n}} (x+3)^n}$$

$$= \frac{-2}{\sqrt[n]{n^2}} (x+3) \quad \therefore |x+3| \leq \frac{1}{2} \Rightarrow R = \frac{1}{2}$$

$$-1 < x+3 < \frac{1}{2}$$

$$-4 < x < -2$$

when $x = -4$

$$\sum \frac{(-2)^n (-1)^n}{\sqrt{n}}$$

$$\sum \frac{2^n}{\sqrt{n}}$$

$$\lim_{n \rightarrow \infty} \frac{2^n}{\sqrt{n}} \neq \frac{1}{\sqrt{n}}$$

$$\lim_{n \rightarrow \infty} 2^n = \infty$$

both diverge
by L.C.T

when $x = -2$

$$\sum \frac{(-2)^n}{\sqrt{n}}$$

Converges conditionally by A.S.T

\Rightarrow the series converges on the interval $(-4, -2]$

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QUESTION FOUR: [16 points]

Consider the integral $\int x \cos(x^3) dx$.

(a) Write down the Maclaurin series for $\cos(x)$, $\cos(x^3)$, and $x \cos(x^3)$.

(b) Evaluate $\int x \cos(x^3) dx$ as an infinite series.

$$\cos(x) = \frac{(-1)^n x^{2n}}{2n!}$$

$$\cos(x^3) = \frac{(-1)^n x^{6n}}{2n!}$$

Maclaurin

$$\cos(x) = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \dots$$

$$\cos(x^3) = 1 + \frac{x^6}{2!} + \frac{x^{12}}{4!} + \frac{x^{18}}{6!} + \dots$$

$$x \cos(x^3) = x + \frac{x^7}{2!} + \frac{x^{13}}{4!} + \frac{x^{19}}{6!} + \dots$$

$$\int_0^1 x \cos(x^3) dx = x + \frac{x^7}{2!} + \frac{x^{13}}{4!} + \frac{x^{19}}{6!} + \frac{x^{25}}{8!} + \dots$$

$$\int_0^1 x \cos(x^3) dx = \frac{2x + x^7}{2} + \dots$$

$$\int_0^1 x \cos(x^3) dx = \frac{2x + x^7}{2} + \dots$$

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Handwritten notes: $\frac{\pi}{2} \times \frac{2\pi}{5}$, $\frac{3\pi}{5}$, $\frac{\pi}{5} \times \frac{4\pi}{5} = \frac{4\pi}{5}$, $\frac{5\pi}{5}$, $\frac{\pi}{5} \times \frac{6\pi}{5}$

Key

Birzeit University
Department of Mathematics
Math 132

Summer/2009
Number: ...
Section:

Final exam
Name :
Instructor:

Q#1 (72%) Circle the correct answer.

$\ln \frac{\pi}{3} \times \frac{6}{\pi}$

1) $\int_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} \frac{dx}{(1+x^2) \tan^{-1} x} =$

- a) $\ln 4 - \ln 3$
- c) $\ln 3 - \ln 2$

- b) $\ln 2$
- d) $\frac{\pi}{12}$

2) If $y = (\ln x)^x$ then $\frac{dy}{dx}$

a) $(\ln x)^x \left(\frac{1}{\ln x} + \ln x \right)$

b) $(\ln x)^{x-1}$

c) $x (\ln x)^{x-1}$

d) $(\ln x)^x \left[\frac{1}{\ln x} + \ln(\ln x) \right]$

3) $\int_1^{e^3} \frac{1}{x \sqrt{1 + \ln x}} dx =$

a) $\ln \frac{4}{3}$

b) 2

c) $\frac{4}{3}$

d) $\ln \frac{3}{2}$

4) $\int_1^2 \frac{\sinh(\ln x)}{x} dx =$

a) 0

b) 1

c) $\frac{1}{4}$

d) $\frac{1}{2}$

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5) if $y = \tan^{-1}\left(\frac{1}{x}\right)$ then $\csc y =$

a) $\frac{\sqrt{1+x^2}}{x}$

b) $\sqrt{1+x^2}$

c) $\frac{x}{\sqrt{1+x^2}}$

d) $\frac{1}{\sqrt{1+x^2}}$

6) if $y = 5^{\ln x}$ then $\frac{dy}{dx}$ when $x=1$ is:

a) 0

b) $-\ln 5$

c) $\ln 5$

d) 1

7) $\int_0^1 e^{\sqrt{x}} dx =$

a) 0

b) 4

c) 3

d) 2

8) $\int x^2 e^{3x} dx =$

a) $\frac{1}{3}x^2 e^{3x} + \frac{2}{9}x e^{3x} + \frac{2}{27}e^{3x} + c$

b) $\frac{1}{3}x^2 e^{3x} + \frac{2}{9}x e^{3x} - \frac{2}{27}e^{3x} + c$

c) $\frac{1}{3}x^2 e^{3x} - \frac{2}{9}x e^{3x} + \frac{2}{27}e^{3x} + c$

d) $-\frac{1}{3}x^2 e^{3x} + \frac{2}{9}x e^{3x} - \frac{2}{27}e^{3x} + c$

9) $\int \frac{dx}{(x+2)\sqrt{x^2+4x}} =$

a) $\sqrt{x^2+4x} + c$

b) $\frac{1}{2} \sec^{-1} \left| \frac{x+2}{2} \right| + c$

c) $\frac{1}{2} \ln |x^2+2x| + c$

d) $\sinh^{-1}(x+2) + c$

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10) If $f(x) = xe^x$, then $(f^{-1})'(e) =$

(a) $\frac{1}{e^e(e+1)}$

(b) $e^e + ee^e$

(c) $\frac{1}{2e}$

(d) None of the above

$x e^x = e^x$

11) Consider the improper integrals:

(i) $\int_3^{\infty} \frac{dx}{(x-3)^2}$

(ii) $\int_{-5}^{\infty} \frac{dx}{\sqrt{x+5}}$

- (a) only integral (i) converge
 (b) both integrals converge

- b) only integral (ii) converges
 (d) both integrals diverge.

12. The power series $x + \frac{x^2}{2} + \frac{x^3}{3} + \dots + \frac{x^n}{n} + \dots$ converges if and only if

(a) $-1 < x < 1$

(c) $-1 \leq x < 1$

$\frac{x}{1-x} = 1 + x + x^2 + \dots + x^{n-1}$

(b) $-1 \leq x \leq 1$

(d) $-1 < x \leq 1$

$(1+x)^{\frac{1}{2}} = 1 + \frac{1}{2}x - \frac{1}{8}x^2$

13. The Maclaurin series of order 3 for $f(x) = \sqrt{x+1}$ is

(a) $1 + \frac{x}{2} - \frac{x^2}{4} + \frac{3x^3}{8}$

(c) $1 - \frac{x}{2} + \frac{x^2}{8} - \frac{x^3}{16}$

(b) $1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16}$

(d) $1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{8}$

14) Determine whether $\int_2^{30} \frac{dx}{(x-3)^{2/3}}$ converges or diverges. If the integral

converges, find its value

a) converges, 15

c) converges, 1

b) converges, 3

d) diverges.

Handwritten work for question 14:

$$\int_2^{30} \frac{dx}{(x-3)^{2/3}} = \int_2^3 \frac{dx}{(x-3)^{2/3}} + \int_3^{30} \frac{dx}{(x-3)^{2/3}}$$

$$= \left[\frac{3\sqrt{x-3}}{1} \right]_2^3 + \left[\frac{3\sqrt{x-3}}{1} \right]_3^{30}$$

$$= 0 - 3\sqrt{-1} + 9 - 0 = 9$$

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15) If $(\frac{1+i}{1-i})^4 + z = 2+i$ then $z =$

a) $2-i$

b) $1-2i$

c) $2+4i$

d) $1+i$

16) The series $\sum_1^{\infty} \frac{n^n}{2^n 3^n}$

a) Diverges by Ratio test

b) Converges by n^{th} term test

c) Converges by Integral test

d) Converges by n^{th} root test

17) The series $\sum_1^{\infty} (-1)^n (\frac{n^2}{2n^4 + 5})$

a) Converges by limit comparison test

b) Diverges by n^{th} term test

c) Converges by n^{th} term test

d) Converges absolutely

18) The sequence $\{a_n\} = \{ \frac{1+(-1)^n}{n} \}$

a) Converges to 1

b) Converges to 0

c) Converges to 2

d) Diverges

19) $\int \sin^{-1} x \, dx =$

a) $x \sin^{-1} x - 2\sqrt{1-x^2} + c$

b) $x \sin^{-1} x + 2\sqrt{1-x^2} + c$

c) $x \sin^{-1} x - \sqrt{1-x^2} + c$

d) $x \sin^{-1} x + \sqrt{1-x^2} + c$

$u = \sin^{-1} x \quad dv = dx$
 $du = \frac{1}{\sqrt{1-x^2}} dx \quad v = x$

$x \sin^{-1} x + \frac{1}{2} \int \frac{-2x \, dx}{\sqrt{1-x^2}}$

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$$20) \int \frac{dx}{\sqrt{2x-x^2}} =$$

a) $2\sqrt{2x-x^2} + c$

c) $\sin^{-1}(x-2) + c$

b) $\sin^{-1}(x-1) + c$

d) $\sec^{-1}(x-1) + c$

21) The series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{(n+1)(n+2)}}$

a) Converge conditionally

b) Converge absolutely

c) Diverges

d) Converges to 2

22) $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$

$\frac{1}{\ln n} > \frac{1}{n}$ ✓

a) Converges Absolutely

b) Converges conditionally

c) Diverges by alternating series theorem

d) Diverges by n^{th} -term test

$e^x = e^x$

23) If $\cosh x + \sinh x = e$ then $x =$

a) e

b) 1

c) $\ln 5$

d) $\ln \sqrt{5}$

24) $\int_3^4 \frac{3dx}{x^2+x-2} =$

a) $\ln \frac{5}{4}$

b) $\ln \frac{4}{5}$

c) $\ln \frac{8}{5}$

d) $\ln \frac{5}{8}$

$\int \frac{1}{x-1} - \frac{1}{x+2}$

$\ln \frac{x-1}{x+2} \Big|_3^4 = \ln \frac{3}{5} \times \frac{5}{2}$

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2017



2016

مَجْلِسُ الطَّلَبَةِ

Q2(9%) Use series to find an estimate for $\int_0^{\frac{1}{2}} \frac{\tan^{-1}x}{x} dx$ with an error of magnitude less than 10^{-3}



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2017



2016

مَجْلَسُ الطَّلَبَةِ

Q3(10%) Solve $z^4 = -81$ in the field of complex numbers



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2016

مَجْلِسُ الطَّلَبَةِ

Q4)(10%) Sketch the graph of the conic section

$$9x^2 + 25y^2 + 18x - 100y = 116 \quad \text{indicating}$$

a) foci

b) vertices

c) eccentricity

d) directrices

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2017



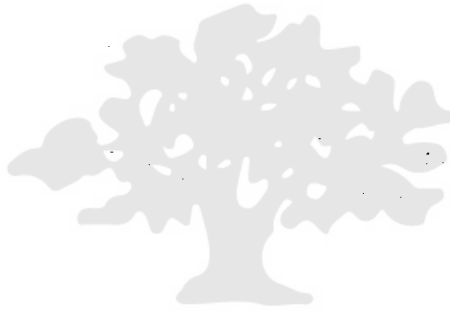
2016

مجلس الطلبة

جَامِعَةُ بِيْرزَيْتِ

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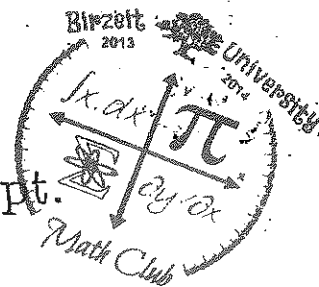
2017



2016

مَجْلَسُ الطَّلَبَةِ

Birzeit University
 Math. & Comp. Science Dept.
 Math. 132



Dr. Marwan Awartani

Final Exam

Fall

Student Name: _____

Number: _____

Section: _____

Q1: (60 points) Circle the MOST correct answer:

1. $\int_e^{e^2} \frac{1}{x \ln x} dx =$

- (a) 1
- (b) $\ln 2$
- (c) $\ln(\frac{1}{2})$
- (d) 0

$u = \ln x$
 $du = \frac{dx}{x}$ $dx = x du$
 $= \int_e^{e^2} \frac{du}{u} = \ln|u| \Big|_e^{e^2} = \ln|\ln x| \Big|_e^{e^2}$
 $= \ln \ln e^2 - \ln \ln e$
 $= \ln 2$

2. The curve with parametric equations $x = \sin t$, $y = \cos t$, $-\infty < t < \infty$.

- (a) a segment of a parabola
- (b) A circle
- (c) An ellipse
- (d) A hyperbola

3. The slope of the curve $x = \sin 2t$, $y = \cos t$ at $t = +\frac{\pi}{6}$ is:

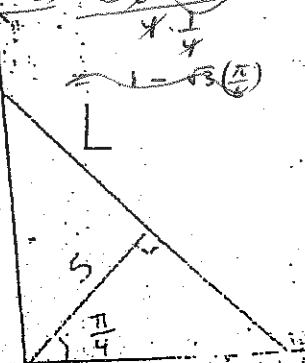
- (a) 0
- (b) 1
- (c) $\frac{1}{2}$
- (d) $-\frac{1}{2}$

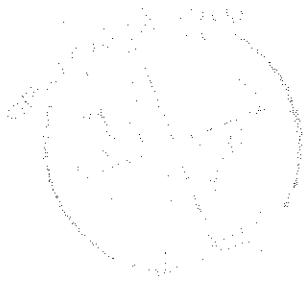
$x = 2 \sin t \cos t$
 $x = 2 \sin \frac{\pi}{6} \cos \frac{\pi}{6} = 2(\frac{1}{2})(\frac{\sqrt{3}}{2}) = \frac{\sqrt{3}}{2}$

$y = \cos t$
 $y = \frac{x}{2 \sin t} \Rightarrow y = \frac{2 \cos t}{4 \sin t} = \frac{2 \cos t}{2 \sin t} = \frac{2 \cos t}{2 \cdot \frac{1}{2}} = 2 \cos t$
 $y = 2 \cos(\frac{\pi}{6}) = 2 \cdot \frac{\sqrt{3}}{2} = \sqrt{3}$

4. The polar equation of straight line l , is:

- (a) $r \cos(\theta + \frac{\pi}{4}) = -5$
- (b) $r \cos(\theta - \frac{\pi}{4}) = 5$
- (c) $r \sin(\theta + \frac{\pi}{4}) = -5$
- (d) $r \cos \theta = 5$





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جامعة بيرزيت

5. The graphs of the curves with polar coordinates $r = \sin \theta$, $r = -\cos \theta$ intersect at:

- (a) Only at the origin.
- (b) Only when $\tan \theta = -1$
- (c) At exactly two points.
- (d) At exactly three points.

6. The equation of $x^2 + 5xy + y^2 = 3$ is

- (a) Circle
- (b) Ellipse
- (c) A hyperbola
- (d) A parabola

7. One of the following is not an improper integral

(a) $\int_0^{10} \frac{\sin x}{x} dx$ $\left(x - \frac{x^3}{3!} \right)$

(b) $\int_{-\infty}^{\infty} x dx$

(c) $\int_0^1 \frac{1}{\sqrt{x}} dx$

(d) $\int_0^{\infty} \frac{dx}{x^2 - 1}$

$du = \cos x \cdot dx$
 $u = \sin x$

8. $\int \sin^2 x \cos^3 x dx = \int \sin^2 x \cos^2 x \cos x dx$
 $= \int u^2 (1-u^2) \cdot du = \int (u^2 - u^4) \cdot du$

(a) $\frac{\sin^3 x \cos^4 x}{12} + c$

(b) $\frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + c$

(c) $\frac{\cos^3 x \sin^4 x}{15} + c$

(d) $\frac{\sin^3 x}{15} (5 + 3 \sin^2 x) + c$

$= \frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + c$

9. If a particle moves on a parametric curve described by $x = t^2$, $y = \sqrt{1-t^4}$, $-1 \leq t \leq 1$, then

- (a) The initial point is (1,0) and the end point is (0,1).
- (b) the motion is clockwise.
- (c) the motion is counter clockwise.
- (d) None of the above.

10. $\lim_{x \rightarrow 0^+} (1+x)^{\frac{1}{x}}$

- (a) e
- (b) 1
- (c) $\frac{1}{e}$
- (d) 0

$u = \frac{1}{x}$
 $du = -\frac{dx}{x^2} \rightarrow dx = -x^2 \cdot du$
 $x \rightarrow 0 \text{ as } u \rightarrow \infty$

$\lim_{u \rightarrow \infty} \left(1 + \frac{1}{u}\right)^u$

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10. The graph of the curves with polar coordinates $r = 1 + \cos \theta$ and $r = 1 - \cos \theta$ intersects in

- (a) One point only
- (b) Two points only
- (c) Three points only
- (d) Four points only

11. The slope of the polar curve $r = 1 + 2 \cos \theta$ at the origin is

- (a) 1
- (b) -1
- (c) $\sqrt{3}$
- (d) $\pm\sqrt{3}$

12. $\lim_{x \rightarrow 0^+} (1+x)^{\frac{1}{x}} =$

- (a) e
- (b) 1
- (c) $\frac{1}{e}$
- (d) 0

Handwritten solution for Q12:
 $u = \tan^{-1} x$
 $x = \tan u$
 $dx = \sec^2 u \cdot du$
 $\lim_{x \rightarrow 0} \frac{\tan^{-1} x}{x} = \lim_{u \rightarrow 0} \frac{u}{\tan u} = \lim_{u \rightarrow 0} \frac{u \cos u}{\sin u}$

13. $\lim_{x \rightarrow 0} \frac{\tan^{-1} x}{x} =$

- (a) 1
- (b) -1
- (c) 0
- (d) ∞

Handwritten solution for Q13:
 $u = \tan^{-1} x$
 $du = \frac{dx}{1+x^2}$
 $dx = (1+x^2) \cdot du$
 $= \lim_{x \rightarrow 0} \frac{(1+x^2)}{x} \cdot dx = \frac{1 + \tan^2 u}{\tan u} \cdot dx$
 $= \int \frac{\sec^2 u}{\tan u} \cdot dx$
 $= \int \frac{1}{\cos^2 u} \cdot \frac{\cos u}{\sin u} \cdot dx = \int \frac{1}{\sin u \cos u} \cdot dx$

14. $7^{7^7} =$

- (a) $\frac{1}{7}$
- (b) $\frac{1}{5}$
- (c) 5
- (d) 7

15. The eccentricity of the conic section $\frac{x^2}{9} - \frac{y^2}{16} = 1$ is

- (a) $\frac{5}{3}$
- (b) $\frac{4}{5}$
- (c) $\frac{3}{4}$
- (d) $\frac{4}{3}$

Handwritten solution for Q15:
 $a = 3$
 $b = 4$
 $c^2 = a^2 + b^2 = 9 + 16 = 25$
 $c = 5$
 $e = \frac{c}{a} = \frac{5}{3}$

16. The length of the polar curve $r = 4 \cos \theta$, $0 \leq \theta \leq \frac{\pi}{2}$ is

- (a) π
- (b) 2π
- (c) 3π

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17. The surface area generated by revolving $r = 4 \cos \theta$ $0 \leq \theta \leq \frac{\pi}{2}$ about x-axis is

- (a) 2π
- (b) 4π
- (c) 8π
- (d) 16π

18. The Cartesian coordinates of the point $P(-4, \frac{\pi}{4})$ is

- (a) $(-2\sqrt{2}, 2\sqrt{2})$
- (b) $(-2\sqrt{2}, -2\sqrt{2})$
- (c) $(2\sqrt{2}, 2\sqrt{2})$
- (d) $(2\sqrt{2}, -2\sqrt{2})$

19. The angle θ that eliminate xy term in $2x^2 + \sqrt{3}xy + y^2 - 2y = 6$ is

- (a) $\frac{\pi}{3}$
- (b) $\frac{\pi}{4}$
- (c) $\frac{\pi}{6}$
- (d) $\frac{\pi}{2}$

20. $\sec^{-1}(-\sqrt{2}) =$ $-\sqrt{2} = \sec y$

21. $\frac{\pi}{4}$ $-\sqrt{2} = \frac{1}{\cos y}$

22. $\frac{-\pi}{4}$ $\cos y = \frac{-1}{\sqrt{2}}$

23. $\frac{3\pi}{4}$ $y = \frac{\pi}{4} + \frac{\pi}{4} = \frac{2\pi}{4}$

(24) $\frac{5\pi}{4}$ $\frac{\pi}{4} - \frac{\pi}{4} = -\frac{2\pi}{4}$

II (15 points): Evaluate the following integrals

$\int \sin^{\frac{5}{2}} x$

$\int \sec x$

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جامعة الزيتونة



Birzeit University
Math. & Comp. Science Dept.
Math. 132

Dr. Hasan Yousef

Final Exam

Summer ~~2016~~

Student Name: _____ Number: _____ Section: _____

I (40points) : Circle the MOST correct answer:

1. If $\sinh x = \frac{-3}{4}$ then $\cosh x =$

- (a) $\frac{3}{4}$
- (b) $\frac{4}{3}$
- (c) $\frac{-3}{5}$
- (d) $\frac{-5}{4}$



2. The conic section with Foci $(\pm 1, 0)$ and vertices $(\pm 2, 0)$ is an

- (a) Ellipse
- (b) Parabola
- (c) Hyperbola
- (d) A circle

$c=1 \quad a=2$
 $a > c$
 $e > 1$

3. The conic section with eccentricity $\frac{1}{2}$ and directrix $x = 2$ has equation

- (a) $2x^2 + y^2 = 1$
- (b) $x^2 - y^2 = 1$
- (c) $y^2 - x^2 = 2$
- (d) $x^2 + \frac{4}{3}y^2 = 1$

$e = \frac{c}{a} = \frac{1}{2} \quad 2 \cdot \frac{1}{2} \quad \frac{a}{e} = 2$
 $\frac{a}{2} = \frac{c}{a} = \frac{1}{2} \quad \leftarrow e = \frac{a}{2}$
 $a^2 = 2c \quad a=1 \quad 2c=a$
 $a=1$

4. The directrix of the parabola $x = \frac{y^2}{2}$ is given by

- (a) $x = 1$
- (b) $y = \frac{1}{2}$
- (c) $y = \frac{1}{2}$
- (d) $x = \frac{1}{2}$

$\frac{x^2}{1} + \frac{y^2}{3} = 1$
 $xy = 2x$
 $4px = 2x$
 $p = \frac{1}{2}$

$a = 2e$
 $= 2 \cdot \frac{1}{2}$
 $= 1$
 $e = ea = \frac{1}{2} \cdot 1$
 $e = \frac{1}{2}$

~~RAE~~

$D = \frac{a}{e} = 2$

$a = 2e$

$e = \frac{c}{a} = \frac{1}{2}$

$a = 1$

$a = 2c \rightarrow c = \frac{1}{2}$

$b^2 = a^2 - c^2$

$= 1 - \frac{1}{4}$
 $b = \frac{\sqrt{3}}{2}$

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5. The conic section $x^2 + 4xy + \sqrt{2}y^2 + 5 = 0$ is

- (a) Ellips
- (b) Parabola
- (c) Hyperbola
- (d) A Circle

$$x^2 + \sqrt{2}y^2 = -4xy - 5$$

$$\frac{x^2 + \sqrt{2}y^2}{-4xy - 5} = \frac{-4xy}{-4xy - 5}$$

$$= \frac{x}{-4y} + \frac{\sqrt{2}y}{-4x} = \frac{5}{-4xy}$$

6. $x = \cos 2t, y = \sin^2 t, 0 \leq t \leq \frac{\pi}{4}$ represents

- (a) half of a circle
- (b) half of an Ellipse
- (c) a line segment
- (d) a parabola

$$-4x^2 - 4\sqrt{2}y^2 = -5$$

7. The slope of the Ellipse $x = 2 \sin t, y = 3 \cos t, 0 \leq t \leq 2\pi$ at the point $(\sqrt{2}, \frac{3}{\sqrt{2}})$ is

- (a) $-\frac{2}{3}$
- (b) 3
- (c) $\frac{3}{2}$
- (d) $\frac{2}{3}$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{4 \sin^2 t}{a^2} + \frac{9 \cos^2 t}{b^2} = 1$$

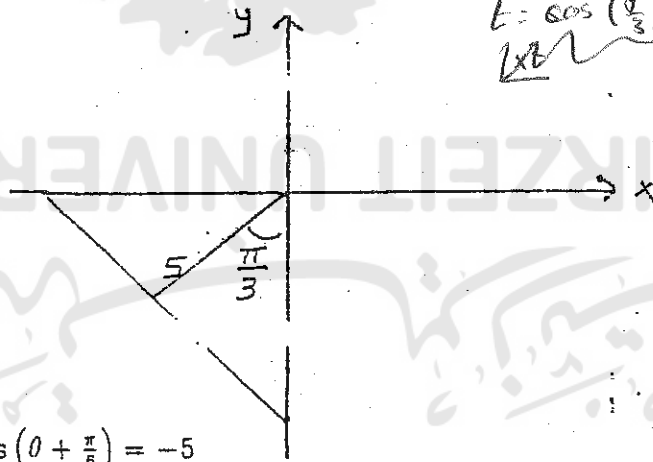
$$t = \sin^{-1}\left(\frac{x}{2}\right) = \frac{\pi}{4}$$

$$t = \cos^{-1}\left(\frac{y}{3}\right) = \frac{\pi}{4}$$

$$\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\frac{3}{3\sqrt{2}} = \frac{1}{\sqrt{2}}$$

8. An equation of the line in the figure is



- (a) $r \cos\left(\theta + \frac{\pi}{6}\right) = -5$
- (b) $r \sin\left(\theta + \frac{2\pi}{3}\right) = -5$
- (c) $r \cos\left(\theta + \frac{5\pi}{6}\right) = 5$
- (d) $r \sin\left(\theta + \frac{\pi}{6}\right) = -5$

9. The polar Equation of the circle with center $P(-2, \frac{\pi}{4})$ and radius 4 is

- (a) $r = -4 \sin\left(\theta + \frac{\pi}{4}\right)$
- (b) $r = -4 \sin\left(\theta + \frac{3\pi}{4}\right)$
- (c) $r = 4 \sin\left(\theta - \frac{3\pi}{4}\right)$
- (d) $r = 4 \sin\left(\theta - \frac{\pi}{4}\right)$

جامعة الزيتونة

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$$\int \frac{x^2 \cdot dy}{2x\sqrt{1+x^2}}$$

$$= \int \frac{x \cdot dy}{2\sqrt{1+x^2}} = \int \frac{\sqrt{u} \cdot dy}{2\sqrt{1+u}}$$

$$u = x^2$$

$$du = 2x \cdot dx$$

$$dx = \frac{du}{2x}$$

Q2: (15 points) (a) $\int \frac{x^2 dx}{\sqrt{1+x^2}} =$

$$u = 1+x^2$$

$$du = 2x \cdot dx$$

$$dx = \frac{du}{2x}$$

$$= \int \frac{x^2 \cdot dy}{2x\sqrt{u}} = \int \frac{\sqrt{u-1} \cdot dy}{2\sqrt{u}}$$

$$= \frac{1}{2} \int \sqrt{\frac{u-1}{u}} \cdot dy = \frac{1}{2} \int \sqrt{1 - \frac{1}{u}} \cdot dy$$

$$u = \sqrt{1+x^2}$$

$$du = \frac{2x \cdot dx}{2\sqrt{1+x^2}}$$

$$= \int \frac{x^2 \cdot \sqrt{1+x^2} \cdot dy}{x \cdot (u)} = \int \sqrt{u^2 - 1} \cdot dy$$

$$du = \frac{x \cdot dx}{\sqrt{1+x^2}}$$

$$= \int (u^2 - 1)^{\frac{1}{2}} \cdot dy = \frac{2}{3} (u^2 - 1)^{\frac{3}{2}}$$

$$dx = \frac{\sqrt{1+x^2}}{x} \cdot dy$$

(b) $\int \frac{x^2 + x + 1}{x^3 + x} dx =$

$$u = \sqrt{1+x^2}$$

$$u^2 = 1+x^2$$

$$x = \sqrt{u^2 - 1}$$

$$\frac{x^2 + x + 1}{x(x^2 + 1)} = \frac{A}{x} + \frac{Bx + C}{x^2 + 1}$$

(c) $\lim_{x \rightarrow \infty} \left(\frac{x-1}{x-2} \right)^x = \lim_{x \rightarrow \infty} \frac{(x-1)^x}{(x-2)^x} = \lim_{x \rightarrow \infty} \frac{x(x-1)^{x-1}}{x(x-2)^{x-1}}$

$$= \lim_{u \rightarrow \infty} \frac{(u)^{u+1}}{(u)^{u+2}}$$

$$= \lim_{u \rightarrow \infty} u^{(u+1)-(u+2)}$$

$$= \lim_{u \rightarrow \infty} u^{-1}$$

$$= \lim_{u \rightarrow \infty} \frac{1}{u} = 0$$

$$\frac{x(x-1)^{x-1}}{x(x-2)^{x-1}}$$

$$u = x-1$$

$$du = dx$$

$$x = u+1$$

$$x \rightarrow \infty \quad u = \infty$$

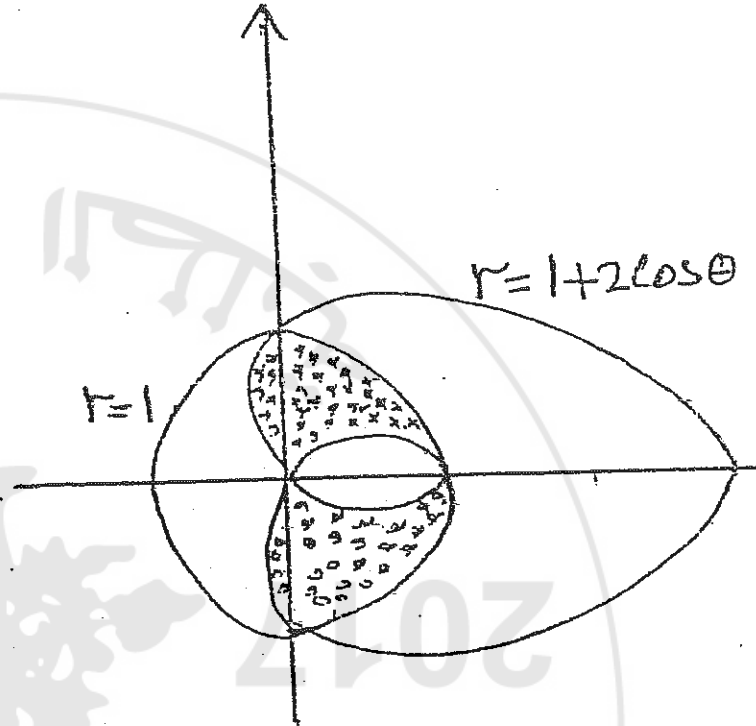
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IV (15 points): Find the area of the shaded region.



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جامعة الزيتونة



Birzeit University
Math. & Comp. Science Dept.
Math. 132

Final Exam

Second Semester

The Name of Discussion Teacher:

Student Name: _____ Number: _____ Section: _____

I (50 points) : Circle the MOST correct answer:

1. If $\sinh x = 3$ then $\cosh(-x) =$

- (a) $\sqrt{10}$
- (b) $-\sqrt{10}$
- (c) $\sqrt{8}$
- (d) $\sqrt{10}, -\sqrt{10}$



2. $\int \frac{ax+b}{x^3+x^2} dx$ is a rational function if

- (a) $b = 0$
- (b) $a = b$
- (c) $a = -b$
- (d) $a = b$

3. If $2^{x^2} = 4 \cdot 2^x$ and $x > 0$ then $x =$

- (a) 2
- (b) -1
- (c) 2, -1
- (d) 4

$$x \ln 2^{2^x} = x \ln 4 \cdot 2$$

$$x = \frac{\ln 4 \cdot 2}{\ln 2} = \ln 2 \cdot 2$$

$$u = e^x$$

$$du = e^x \cdot dx$$

$$x=1 \rightarrow u=e$$

$$x=0 \rightarrow u=1$$

4. $\int_0^1 \frac{e^x - e^{-x}}{e^x + e^{-x}} dx =$

- (a) $\tanh^{-1}(1)$
- (b) $\ln(e^2 + 1) - \ln 2$
- (c) $\ln(e^2 + 1) - \ln 2 - 1$
- (d) None of the above

$$= \int_0^1 \frac{e^x - e^{-x}}{e^x + e^{-x}} dx = \int_1^e \frac{u - \frac{1}{u}}{u + \frac{1}{u}} \cdot \frac{1}{u} du = \int_1^e \frac{u^2 - 1}{u(u^2 + 1)} du$$

Handwritten work for question 4:

$$\frac{u^2 - 1}{u(u^2 + 1)} = \frac{A}{u} + \frac{B}{u^2 + 1}$$

$$u^2 - 1 = A(u^2 + 1) + B(u)$$

$$A = 1, \quad u = 0 \Rightarrow B = -2$$

$$2A + B = 0 \Rightarrow B = -2$$

$$B = -2$$

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$$= 2x \ln x - \frac{2(u e^u - e^u)}{\ln 3}$$

$$f(x) = u \quad g(x) = e^u$$

$$f'(x) = 1 \quad \int g(x) = e^u$$

$$\int \int g(x) = e^u$$

$$= \int \frac{\ln x^2}{\ln 3} dx = \frac{1}{\ln 3} \int \ln x^2 dx$$

$$5. \int \log_3 x^2 dx = \frac{1}{\ln 3} \int u \cdot \frac{x}{2} dx \quad u = \ln x^2$$

$$du = \frac{2x}{x^2} dx$$

(a) $2x \log_3 x^2 - x + c$

(b) $2x \log_3 x - x + c$

(c) $\frac{2}{\ln 3} (x \ln x + x) + c$

(d) $\frac{2}{\ln 3} (x \ln x - x) + c$

$$\frac{1}{2 \ln 3} \int u \cdot e^u du$$

$$z = e^u$$

$$dz = e^u du$$

$$u = \frac{z^2}{2}$$

$$du = \frac{2}{x} dx$$

$$dx = \frac{x}{2} du$$

$$\frac{u}{2} = \ln x$$

6. If $\int_0^b f(x) dx$ diverges and $\int_a^b g(x) dx$ diverges then $\int_a^b f(x)g(x) dx$

(a) converges always

(b) diverges always

(c) Can't decide

(d) Converges if $f(x) = g(x)$

$$= \frac{e^4 u^2}{2} - \int \frac{u^2}{2} e^4 du$$

7. A line $y = ax + b$ and a curve of a function $y = e^x$ can intersect in at most

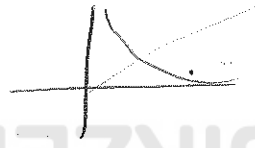
(a) 1 point

(b) 2 points

(c) 3 points

(d) 4 points

$$ax + b = e^x$$



8. If $\sin^{-1} x = \frac{\pi}{3}$ then $\cos^{-1} x =$

(a) $\frac{2\pi}{3}$

(b) $\frac{5\pi}{6}$

(c) $\frac{\pi}{4}$

(d) $\frac{\pi}{3}$

$$x = \frac{\sqrt{3}}{2}$$

$$x = \sin u$$

$$u = \sin^{-1} x$$

$$du = \frac{dx}{\sqrt{1-x^2}}$$

$$dx = \sqrt{1-x^2} du$$

$$= \cos u du$$

$$\frac{x=1}{u=\pi/4} \quad \frac{x=0}{u=0}$$

$$9. \int_0^{\pi/2} \sin x dx + \int_0^{\pi/2} \sin^{-1} x dx = \left[-\cos x \right]_0^{\pi/2} + \left[\sin u \right]_0^{\pi/4}$$

(a) π

(b) $\frac{\pi}{2}$

(c) 1

(d) $\frac{1}{2}$

$$= 0 + 1 + \frac{1}{\sqrt{2}} - 0$$

$$\left(\frac{2+1}{\sqrt{2}} \right) \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2+\sqrt{2}}{2}$$

10. One of the following is not an improper integral.

(a) $\int_0^{10} \frac{\sin x}{x} dx$

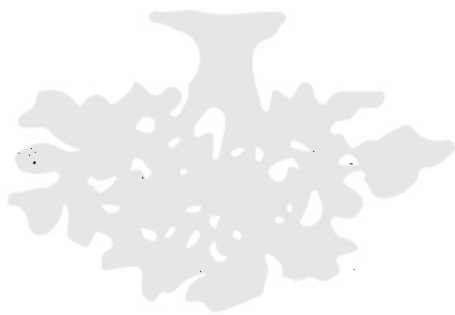
(b) $\int_{-\infty}^a x dx$

(c) $\int_0^1 \frac{1}{\sqrt{x}} dx$

(d) $\int_{-\infty}^{\infty} \frac{dx}{x^2-1}$

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11. $\int \sin^2 x \cos^3 x dx =$

(a) $\frac{\sin^3 x \cos^4 x}{12} + c$

(b) $\frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + c$

(c) $\frac{\cos^3 x \sin^4 x}{12} + c$

(d) $\frac{\sin^3 x}{15} (5 + 3 \sin^2 x) + c$

12. The function $f(x) = \ln|x + 2|$ has domain

(a) $(-2, \infty)$

(b) $(0, \infty)$

(c) $[-1, \infty)$

(d) $[0, \infty)$

13. $\int x^3 e^x dx =$

(a) $\frac{x^4}{4} e^x + c$

(b) $\frac{x^4}{4} e^x - e^x + c$

(c) $e^x(x^3 - 3x^2 + 6x - 6) + c$

(d) $e^x(-x^3 + 3x^2 - 6x + 6) + c$

14. If a particle moves on a parametric curve described by $x = t^2, y = \sqrt{1-t^2}$, $t \leq 1$, then

(a) The initial point is $(1, 0)$ and the end point is $(0, 1)$.

(b) The motion is clockwise.

(c) The motion is counter clockwise.

15. The total distance travelled by a particle moving on the curve $r = 2 \sin \theta$ from $\theta = 0$ to $\theta = 2\pi$ is

(a) 4π

(b) 2π

(c) π

(d) 8π

16. The slope of the tangent lines to the curve $r = 1 - 2 \cos \theta$ at the origin are

(a) 0

(b) $\pm\sqrt{3}$

(c) ± 1

(d) not defined

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$$y = \pm \frac{b}{a} x$$

$$a = 4 \quad c = 5 \\ b = 3$$

$$e = \frac{c}{a} = \frac{5}{4}$$

$$x = \frac{a}{e}$$

17. The directrices of the hyperbola $\frac{(x-2)^2}{16} - \frac{(y-1)^2}{9} = 1$ are

- (a) $y = \pm \frac{16}{5}$
- (b) $x = \pm \frac{16}{5}$
- (c) $x = \frac{-26}{5}, x = \frac{6}{5}$
- (d) $y = \frac{-26}{5}, y = \frac{6}{5}$

$$D = \frac{a}{e} = \frac{4}{\frac{5}{4}} = \frac{16}{5}$$

$$\frac{4}{\frac{5}{4}}$$

18. The polar curves $r = \cos 2\theta, r = \frac{1}{2}$ intersects in

- (a) 1 point
- (b) 2 points
- (c) 4 points
- (d) 8 points

$$x = \pm \frac{16}{5} \\ x = +\frac{16}{5} - 2 \quad x = -\frac{16}{5} - 2$$

19. The circles $x^2 + y^2 = 1$ and $(x-1)^2 + y^2 = a^2$ intersect in two point if:

- (a) $a = 2$
- (b) $2 < a$
- (c) $a < 2$
- (d) $0 < a < 2$

$$x \rightarrow 0 \\ u \rightarrow \infty$$

20. $\lim_{x \rightarrow 0^+} \frac{\ln x}{\sqrt{x}} =$

- (a) 0
- (b) $-\infty$
- (c) ∞
- (d) 1

$$u = \ln x \\ du = \frac{dx}{x}$$

$$dx = x \ln x$$

$$x = e^4 \quad u = \sqrt{x}$$

$$= \frac{1}{x} = \frac{2\sqrt{x}}{x} = \infty$$

77
e0

II (15 points): $\lim_{x \rightarrow \infty} \left(\frac{x-1}{x-2}\right)^x$

$$= \lim_{x \rightarrow \infty} \frac{(x-1)^x}{(x-2)^x} = \frac{x \ln(x-1)}{x \ln(x-2)} = \frac{x - \frac{x^2}{2} + \frac{x^3}{3} \dots}{\ln 2 - \frac{x}{2} - \frac{x^2}{4} \dots}$$

$$f(x) = \ln(x-2) \quad f(0) = \ln 2$$

$$f'(x) = \frac{1}{x-2} \quad f'(0) = -\frac{1}{2} = \ln 2$$

$$f''(x) = -\frac{1}{(x-2)^2} \quad f''(0) = -\frac{1}{4}$$



2. Test for convergence $\int_{-\infty}^{\infty} \frac{dx}{e^x + e^{-x}}$.

$$= \int_{-\infty}^{\infty} \frac{dy}{u(u+\frac{1}{u})} = \int_{-\infty}^{\infty} \frac{du}{u(u^2+1)}$$

$$= \int_{-\infty}^{\infty} \left[\tan^{-1} u \right]_{-\infty}^0 + \left[\tan^{-1} \frac{1}{u} \right]_0^{\infty}$$

$$= \left[\tan^{-1} e^x \right]_{-\infty}^0 + \left[\tan^{-1} (e^x) \right]_0^{\infty}$$

$$= \frac{\pi}{4} - \frac{\pi}{4} + \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

$$u = e^x$$

$$du = e^x \cdot dx$$

$$dx = \frac{du}{e^x} = \frac{du}{u}$$

3. Evaluate $\int \frac{\sqrt{9-x^2}}{x} dx$.

$$\int \frac{\sqrt{4-u}}{2(9-u)} du$$

$$= \int \frac{\sqrt{9-u}}{2u} du$$

$$u = 9 - x^2$$

$$du = -2x \cdot dx$$

$$dx = \frac{du}{-2x}$$

$$u = x^2$$

$$du = 2x \cdot dx$$

$$dx = \frac{du}{2x}$$

~~$a^2 - x^2$~~ $x = a \sin \theta$

$a^2 - x^2$ $x = a \sin \theta$
 $dx = a \cos \theta \cdot d\theta$

$$= \int \frac{\sqrt{a^2 - a^2 \sin^2 \theta}}{a \sin \theta} \cdot a \cos \theta \cdot d\theta$$

$$= \int \frac{\cos^2 \theta \cdot d\theta}{\sin \theta} = \int \cot \theta \csc \theta \cdot d\theta = \csc \theta + C$$

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III. (20 points): (a) Sketch the graph of $r = \frac{8}{2 - \sin \theta}$ and indicate the center, vertices, foci and directrices.

(b) Use part (a) to sketch the graph of $r = \frac{8}{2 - \sin(\theta + \frac{\pi}{4})}$

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IV (20 points): (a) Sketch the graph of the conic section $r = \frac{8}{2 - 4 \cos \theta}$ indicating vertices foci and directrices in polar coordinates.



(b) Use (a) to sketch the graph of $r = \frac{8}{2 - 4 \cos (\theta + \frac{\pi}{4})}$ indicating vertices, foci and directrices in polar coordinates.

بیتنا الحکیم

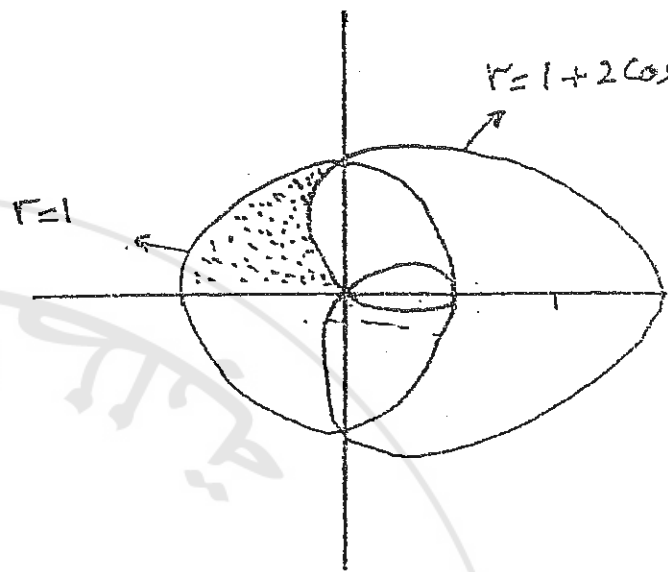
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V (15 points): Find the shaded area.



IV (20 points): (a) Sketch the graph of the conic section $r = \frac{8}{2 - 4 \cos \theta}$ indicating vertices foci and directrices in polar coordinates.

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جامعة الزيتونة

Birzeit University- Mathematics Department
Math 132

Dr. Marwan Alogqili (Sec.2) and Dr. Marwan Awartani (Sec.1&3)

Third Exam

Fall 2002/2003

Name: Ali Nagef Tagatga

Number: C.1.1.2.73

There are 10 (T/F) questions and 14 multiple choice. Calculators are not allowed.

Question 1 Circle the most correct answer:

$\lim_{n \rightarrow \infty} \frac{n}{n+10^6} = \frac{1}{1} = 1$

70

1. $\sum_{n=1}^{\infty} \frac{1}{n+(10)^n}$

- (a) Converges to 0.
- (b) Converges by nth term test.
- (c) Diverges.
- (d) None.

2. One of the following p-series converges:

- (a) $\sum_{n=1}^{\infty} \frac{1}{n^{1.1}}$
- (b) $\sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$
- (c) $\sum_{n=1}^{\infty} \frac{1}{n}$
- (d) $\sum_{n=1}^{\infty} \frac{1}{n^{1/4}}$

$p > 1$

3. The sequence $a_n = (1 - \frac{1}{n^2})^n$

- (a) Diverges.
- (b) Converges to -1.
- (c) Converges to 1.
- (d) None of the above.

$\lim_{n \rightarrow \infty} (1 - \frac{1}{n^2})^n$
 $\ln f(x) = n \ln(1 - \frac{1}{n^2})$
 $= n(-\frac{1}{n^2} + \frac{1}{2n^4} - \dots)$
 $= -\frac{1}{n} + \frac{1}{2n^3} - \dots$
 $\rightarrow 0 + 0 = 0$
 $e^0 = 1$

$\lim_{n \rightarrow \infty} (1 - \frac{1}{n^2})^n$

$\ln f(x) = n \ln(1 - \frac{1}{n^2})$
 $= \frac{1}{1/n} \cdot 0$
 $= \frac{0}{1/n^2} = 0$

4. $\sum_{n=1}^{\infty} \tan^{-1}(n+1) - \tan^{-1} n$

- (a) $\pi/2$.
- (b) $\pi/4$.
- (c) $-\pi/4$.
- (d) None of the above.

$\tan^{-1}(2) - \tan^{-1}(1) + \tan^{-1}(3) - \tan^{-1}(2)$
 $= \tan^{-1}(3) - \tan^{-1}(1)$

$\lim_{n \rightarrow \infty} \frac{1}{(1 - \frac{1}{n^2})^n}$
 $= \lim_{n \rightarrow \infty} \frac{-1/n}{1 - \frac{1}{n^2}}$
 $= \lim_{n \rightarrow \infty} \frac{-1/n}{1 - \frac{1}{n^2}} = 0$

5. $\sum_{n=2}^{\infty} \frac{\ln n}{n}$

- (a) Diverges by integral test.
- (b) Diverging by directly comparing it with $\sum \frac{1}{n}$.
- (c) Diverging by limit comparison test with $\sum \frac{1}{n}$.
- (d) All of the above.

$\lim_{n \rightarrow \infty} \ln f(x) = 0$
 $\Rightarrow \ln f(x) = 1$
 $\Rightarrow \lim_{n \rightarrow \infty} f(x) = e^1 = e$

$\frac{\ln n}{n} > \frac{1}{n}$
 $\lim_{n \rightarrow \infty} \frac{\ln n}{n} = 0$

$\int \frac{\ln n}{n} = \int \frac{u}{x} x du$
 $= \int \frac{u^2}{2} du$

$(\tan^{-1} 2 - \tan^{-1} 1) + (\tan^{-1} 3 - \tan^{-1} 2) + \dots + (\tan^{-1} n - \tan^{-1} (n-1))$
 $= \tan^{-1} n - \tan^{-1} 1$
 $\rightarrow \frac{\pi}{4}$

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$$\frac{1 + \cos^2 n}{n^2} < \frac{2}{n^2}$$

6. $\sum_{n=1}^{\infty} \frac{1 + (\cos n)^2}{n^2}$

- (a) Converges.
- (b) Diverges.
- (c) Diverges by the nth term test.
- (d) Cannot determine.

7. $\sum_{n=1}^{\infty} e^{-n} = \frac{1}{e} + \frac{1}{e^2} + \frac{1}{e^3} + \dots$

- (a) $\frac{c}{c-1}$
- (b) $\frac{1}{e-1}$
- (c) $\frac{1}{1-c}$
- (d) None of the above.

Handwritten work for Q7: $(\frac{1}{e})^n$
 $\frac{1}{e^n} = (\frac{1}{e})^n$
 $\frac{1}{1 - \frac{1}{e}} = \frac{1}{\frac{e-1}{e}} = \frac{e}{e-1}$ as $\frac{1}{e} = \frac{1}{e}$
 $\frac{1/e}{e(1 - 1/e)} = \frac{1}{e-1}$

8. One of the following is true:

- (a) $\sum_{n=1}^{\infty} \frac{3^n}{n^{3/2}}$ diverges by ratio test.
- (b) $\sum_{n=1}^{\infty} \frac{1}{n+1}$ converges by nth root test. \times
- (c) $\sum_{n=1}^{\infty} 2^n$ converges. \times
- (d) $\sum_{n=1}^{\infty} \frac{1}{n^n}$ is a geometric series. \times

Handwritten work for Q8(a):
 $\lim_{n \rightarrow \infty} \frac{3^{n+1}}{(n+1)^{3/2}} \cdot \frac{n^{3/2}}{3^n}$
 $= \lim_{n \rightarrow \infty} \frac{n^3}{(n+1)^3} = \frac{3}{2}$

9. The nth partial sum of the series $\sum_{n=1}^{\infty} (\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}})$ is

- (a) $s_n = 1 - \frac{1}{\sqrt{2}}$
- (b) $s_n = \frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}}$
- (c) $s_n = 1 - \frac{1}{\sqrt{n+1}}$
- (d) None of the above.

Handwritten work for Q9:
 $\frac{1}{\sqrt{1}} - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}} + \dots + (\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}})$
 $= 1 - \frac{1}{\sqrt{n+1}}$

10. If $a_1 = 2, a_{n+1} = \frac{2}{n} a_n$ then $\sum_{n=2}^{\infty} a_n$

- (a) Converges by ratio test.
- (b) Diverges.
- (c) Converges by integral test.
- (d) None of the above.

Handwritten work for Q10:
 $2 + 2(2) + 1(4) + \dots$
 $\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} = \frac{\frac{2}{n} a_n}{a_n} = \frac{2}{n} = 0$

11. The series $\sum_{n=0}^{\infty} (\ln x)^n$ converges if

- (a) $-1 < x < 1$
- (b) $-e < x < e$
- (c) $e^{-1} < x < e$
- (d) None.

Handwritten work for Q11:
 $\sum_{n=0}^{\infty} (\ln x)^n$
 $a_1 = (\ln x)^0 = 1$
 $a_2 = (\ln x)^1$
 $a_3 = (\ln x)^2 \Rightarrow r = (\ln x)$

Handwritten work for Q11 (continued):
 $s_n = \frac{1}{1 - (\ln x)}$

Handwritten work for Q11 (continued):
 $e^{-1} < \ln x < 1$

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12. One of the following series converges:

- (a) $\sum_{n=1}^{\infty} n^2$.
- (b) $\sum_{n=1}^{\infty} \frac{n+1}{n}$.
- (c) $\sum_{n=1}^{\infty} \frac{1}{2^n}$.
- (d) $\sum_{n=1}^{\infty} \ln(1/n)$.

- 5 5

13. $\sum_{n=1}^{\infty} \frac{-5}{n}$

- (a) Converges to 0.
- (b) Converges to 1.
- (c) Diverges.
- (d) None of the above.

14. $\sum_{n=1}^{\infty} \frac{2^n}{n^2}$

- (a) Converges.
- (b) Diverges.
- (c) Converges to 0.
- (d) None.

$$\lim_{n \rightarrow \infty} \frac{2^{n+1}}{(n+1)^2} = \lim_{n \rightarrow \infty} \frac{2 \cdot 2^n}{n^2 + 2n + 1} = 2 > 1 \Rightarrow \text{div.}$$

$$\frac{(-1)^{n+2}}{(-1)^{n+1}} = (-1)^{n+2-n-1} = (-1)^1 = -1$$

Question 2. Answer by True or False:

1. The series $\sum_{n=1}^{\infty} (-1)^{n+1}$ diverges.
2. If $\lim_{n \rightarrow \infty} (a_n)^{1/n} = 1$ then $\sum_{n=1}^{\infty} a_n$ converges.
3. Any increasing sequence and bounded from above converges.
4. $\lim_{n \rightarrow \infty} \frac{x^n}{n!} = 0$.
5. The sequence $a_n = 1 + (-1)^n$ diverges.
6. If $\lim_{n \rightarrow \infty} a_n = 0$ then $\sum_{n=1}^{\infty} a_n$ converges.
7. The series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is called the harmonic series.
8. The sequence $a_n = \frac{3n+1}{n+1}$ is nondecreasing.
9. The series $\sum_{n=1}^{\infty} (\sin x)^n$ converges for any value of x .
10. If $\sum_{n=1}^{\infty} a_n$ converges then $\sum_{n=1}^{\infty} k a_n$, k is any constant, converges.

- (true) ✓
- (false) ✓
- (true) ✓
- (true) ✓
- (true) ✓
- (false) ✓
- (false) ✓
- (true) ✓
- (false) ✓
- (true) ✓

Handwritten work for question 10:

$$\lim_{n \rightarrow \infty} \frac{(-1)^{n+2}}{(-1)^{n+1}} = \lim_{n \rightarrow \infty} (-1)^{n+2-n-1} = \lim_{n \rightarrow \infty} (-1)^1 = -1$$

0, 1, 0, 1

$$\sum_{n=2}^{\infty} (-1)^n = \frac{(-1)^2}{1 - (-1)} = \frac{1}{2}$$

~~Handwritten scribbles and numbers at the bottom of the page.~~

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Birzeit University
Math. & Comp. Science Dept.
Math: 132

Third Hour Exam

Second Semester

The Name of Discussion Teacher:

Student Name: _____ Number: _____ Section: _____

I (40 points): Circle the MOST correct answer:

1. The directrix of the parabola $y = 2x^2 + 4x + 1$ is

(a) $y = -\frac{1}{8}$

(b) $y = \frac{7}{8}$

(c) $y = -\frac{9}{8}$

(d) $y = \frac{9}{8}$

2. A circle of radius a and an ellipse of major semi-axis a both centered at the origin meets in

(a) 1 points

(b) 1 points

(c) 2 point

(d) 0 point

3. The quadratic equation $x^2 + 2xy + y^2 - 2x + 2y + 10 = 0$ represents

(a) Parabola

(b) Ellipse

(c) Hyperbola

(d) Circle

4. A parametrization of the line segment with initial point $(0, 1)$ and terminal point $(1, 0)$ is

(a) $x = \sin^2 t$, $y = \cos^2 t$, $0 \leq t \leq \frac{\pi}{2}$

(b) $x = 1 - t$, $y = t$, $0 \leq t \leq 1$

(c) $x = t$, $y = 1 + t$, $-1 \leq t \leq 0$

(d) $x = 1 + t$, $y = t$, $0 \leq t \leq 1$

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5. The Parametrization $x(t) = -\cos t$, $y(t) = \sin t$, $0 \leq t \leq 2\pi$

- (a) A circle with initial point $(1,0)$ counterclockwise.
- (b) A circle with initial point $(-1,0)$ counterclockwise.
- (c) A circle with initial point $(-1,0)$ clockwise.
- (d) A circle with initial point $(1,0)$ clockwise.

6. The length of the curve $x(t) = \sin t$, $y(t) = 1 + \cos t$, $0 \leq t \leq \frac{\pi}{2}$ is:

- (a) π
- (b) 2π
- (c) $\frac{\pi}{4}$
- (d) $\frac{\pi}{2}$

7. The slope of the tangent line to the curve $y(t) = t^2 - \sin t$, $x(t) = \cos t$, at $t = \frac{\pi}{2}$ is:

- (a) $\frac{\pi^2}{2}$
- (b) $-\frac{1}{\pi}$
- (c) -2
- (d) $-\pi$

8. Which one of the following points lies on the curve $r = \cos 2\theta$

- (a) $(0,0)$
- (b) $(1, \frac{\pi}{2})$
- (c) $(\frac{1}{2}, \frac{\pi}{3})$
- (d) all

9. The graph of $r = 2 \csc \theta$ is

- (a) Circle
- (b) Hyperbola
- (c) Hyperbola
- (d) Parabola

10. The curves $\theta = \frac{\pi}{2}$ and $r = 0$

- (a) never meets.
- (b) intersect in one point.
- (c) intersect in infinitely many points.
- (d) are identical.

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11. $7^{\log_7 5} =$

- (a) $\frac{5}{7}$
- (b) $\frac{7}{5}$
- (c) 5
- (d) 7

12. The directrix of the parabola $y = 2x^2 + 4x + 1$ is

- (a) $y = -\frac{1}{8}$
- (b) $y = \frac{7}{8}$
- (c) $y = -\frac{9}{8}$
- (d) $y = \frac{9}{8}$

13. The quadratic equation $x^2 + 2xy + y^2 - 2x + 2y + 10 = 0$ represents

- (a) Parabola
- (b) Ellipse
- (c) Hyperbola
- (d) Circle

14. A parametrization of the line segment with initial point (0,1) and terminal point (1,0) is

- (a) $x = \sin^2 t$, $y = \cos^2 t$, $0 \leq t \leq \frac{\pi}{2}$
- (b) $x = 1 - t$, $y = t$, $0 \leq t \leq 1$
- (c) $x = 1$, $y = 1 + t$, $-1 \leq t \leq 0$
- (d) $x = 1 + t$, $y = 1$, $0 \leq t \leq 1$

15. $\frac{\log_2(x)}{\log_8(x)} =$

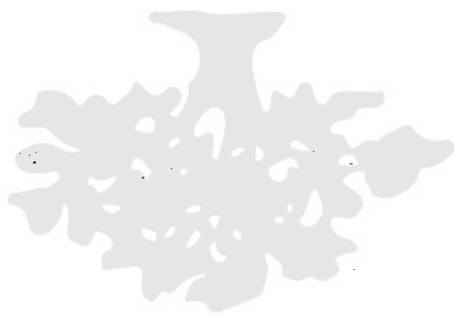
- (a) $\frac{2}{8}$
- (b) $\frac{1}{3}$
- (c) \log_4^x
- (d) 3

16. $\sec^{-1}(-2)$

- (a) $\frac{2\pi}{3}$
- (b) $-\frac{\pi}{3}$
- (c) $-\frac{2\pi}{3}$
- (d) $\frac{3\pi}{4}$

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17. $\lim_{x \rightarrow 0^+} (\ln x - \ln(\sin x))$

- (a) 0
- (b) -1
- (c) +1
- (d) Doesn't exist.

18. $\lim_{x \rightarrow -\infty} (\ln x)^{\frac{1}{2}}$

- (a) ∞
- (b) 0
- (c) 1
- (d) Doesn't exist

19. $\lim_{x \rightarrow \frac{\pi}{2}^-} \frac{\sec x}{\tan x} =$

- (a) 0
- (b) 1
- (c) ∞
- (d) Doesn't exist

20. The order of the functions $x^2, e^x, \ln x$ from slowest growing to fastest growing as $x \rightarrow 0$ is

- (a) $x^2, \ln x, e^x$
- (b) $e^x, x^2, \ln x$
- (c) $x^2, e^x, \ln x$
- (d) $\ln x, x^2, e^x$

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II (20 points): Suppose the path of a moving particle in a plane is described by

$$\begin{aligned}x(t) &= 3 + 4 \sin(t) \\y(t) &= 2 + 5 \cos(t), \quad 0 \leq t \leq \pi\end{aligned}$$

1. Sketch the path of motion and determine the direction.
2. Find the equation of the tangent line at $t = \frac{\pi}{3}$.

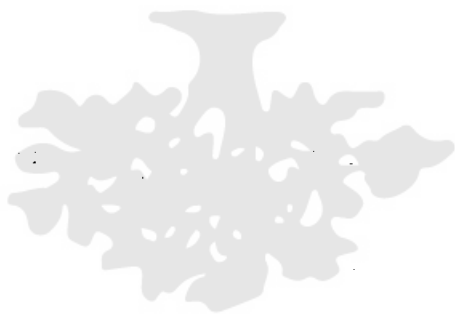
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Question #4:

Graph the conic section $2x^2 + 3xy + 2y^2 - 1 = 0$ in the xy -plane indicating the center, the vertices and the foci in the xy coordinates.

$$2x^2 + 3xy + 2y^2 - 1 = 0$$

$$B^2 - 4AC = 9 - 4(2)(2) < 0 \text{ ellips}$$

$$\cot 2\alpha = \frac{A-C}{B} = \frac{2-2}{3} = 0$$

$$2\alpha = \frac{\pi}{2}$$

$$\alpha = \frac{\pi}{4}$$

$$x = x' \cos \frac{\pi}{4} - y' \sin \frac{\pi}{4}$$

$$y = \frac{x'}{\sqrt{2}} + \frac{y'}{\sqrt{2}}$$

$$y = x' \sin \frac{\pi}{4} + y' \cos \frac{\pi}{4}$$

$$= \frac{x'}{\sqrt{2}} + \frac{y'}{\sqrt{2}}$$

Equation became

$$2 \left(\frac{x' - y'}{\sqrt{2}} \right)^2 + 3 \left(\frac{x' - y'}{\sqrt{2}} \right) \left(\frac{x' + y'}{\sqrt{2}} \right) + 2 \left(\frac{x' + y'}{\sqrt{2}} \right)^2 - 1 = 0$$

$$2 \frac{(x'^2 + y'^2 - 2x'y')}{2} + 3 \left(\frac{x'^2 - y'^2}{2} \right) + 2 \frac{(x'^2 + y'^2 + 2x'y')}{2} - 1 = 0$$

$$x'^2 + y'^2 + \frac{3x'^2}{2} - \frac{3y'^2}{2} + x'^2 + y'^2 = 1$$

$$\frac{7}{2}x'^2 + \frac{1}{2}y'^2 = 1$$

$$\frac{x'^2}{\frac{2}{7}} + \frac{y'^2}{2} = 1$$

~~$$\frac{x'^2}{\frac{2}{7}} + \frac{y'^2}{2} = 1$$~~

Foci

~~$$c = \sqrt{a^2 - b^2}$$~~

$$= \sqrt{\frac{14}{49} - \frac{4}{49}} = \sqrt{\frac{10}{49}} = \frac{\sqrt{10}}{7} \approx 0.45$$

$$c = \sqrt{a^2 - b^2}$$

$$= \sqrt{2 - \frac{2}{7}} = \sqrt{1.714} = 1.3$$

Foci at $x'y'$ plan = $(0, \pm 1.3)$

at xy plan

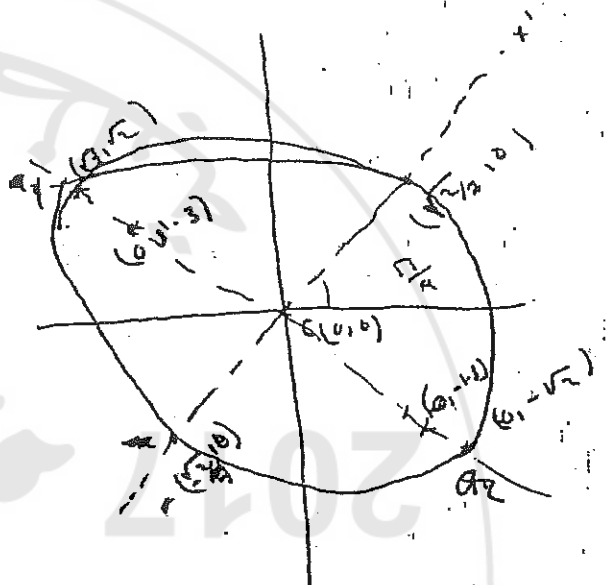
$$y = x' \sin \frac{\pi}{4} + y' \cos \frac{\pi}{4}$$

$$= 0 + 1.3 \cos \frac{\pi}{4}$$

$$= \frac{1.3}{\sqrt{2}}$$

$$x = x' \cos \frac{\pi}{4} - y' \sin \frac{\pi}{4}$$

Foci in xy plan = $(\pm \frac{1.3}{\sqrt{2}}, \pm \frac{1.3}{\sqrt{2}})$



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~~vertices~~
vertices

$$a = (0, \pm\sqrt{2})$$

$$x = x' \cos \alpha - y' \sin \alpha$$

$$x = 0 - \frac{\sqrt{2}}{\sqrt{2}} = -1$$

3 Vertices in Ryzplang

~~$a_1 = (1, 1)$~~

$$a_1 = (-1, 1)$$

$$a_2 = (1, -1)$$

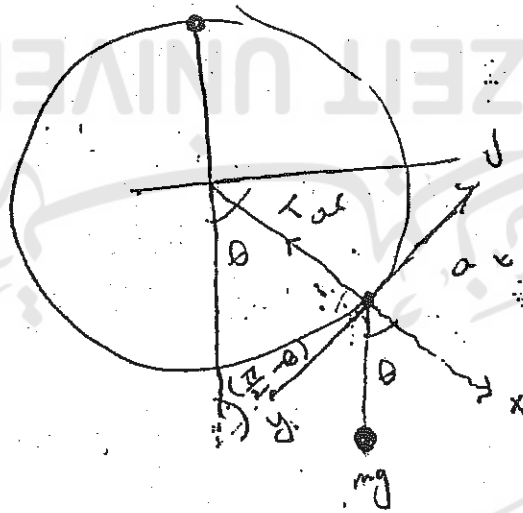
$$y = x' \sin \alpha + y' \cos \alpha$$

$$= 0 + \frac{\sqrt{2}}{\sqrt{2}} = 1$$



$$\frac{mv^2}{R} = \frac{T}{\cos \theta}$$

$$v = \sqrt{Rg \tan \theta}$$



$$\frac{mv^2}{R}$$

$$a = -mg \sin \theta$$

$$mg \cos \theta + \frac{mv^2}{R} = T \quad (1)$$

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Q#7:

Find the series radius of convergence . For what values of x does the series converge :

a) Conditionally

b) absolutely

(i) $\frac{(x-2)^n}{n}$

(ii) $\frac{(-1)^n x^n}{n!}$

Q#8:

a) Find the maclurin series for $f(x) = \ln(1+x^2)$.

b) How many terms are supposed to be used to get an estimate $f(0.2)$ with error less that 10^{-6} .

#9:

Evaluate: $\sum_{n=1}^{\infty} nx^n$ if $|x| < 1$

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MATH DEPARTMENT

MATH 132 TEST THREE

TIME: 60 Min.

JANUARY 2008

NAME: George Hannouch

NUMBER: 106 15 15

SECTION: 1

Instructor's name: Dr. Rimon Jad'ouh

82

QUESTION ONE: (MULTIPLE CHOICE) [40 POINTS]
CIRCLE THE RIGHT ANSWER:

28
30
10
14
82

1. Let $f(x) = \sum_{n=0}^{\infty} x^n$. The interval of convergence of the definite integral 0 to x,

$\int_0^x f(t) dt$ is

- (A) $x = 0$ only
- (B) $|x| \leq 1$
- (C) $-\infty < x < \infty$
- (D) $-1 \leq x < 1$
- (E) $-1 < x < 1$

$$\frac{x}{n+1} = \frac{x^{n+1}}{x+1} - \frac{x^n}{x+1}$$

$$\ln|1+x|$$

2. The coefficient of x^4 in the Maclaurin series for $f(x) = e^{-x/2}$ is

- (A) $-1/24$
- (B) $1/24$
- (C) $1/96$
- (D) $-1/384$
- (E) $1/384$

$$\frac{(-1)^n}{2^{n+1}} \frac{x^n}{n!}$$

$$\frac{1}{e^{x/2}}$$

$$\frac{1}{4 e^{x/2}}$$

3. Which of the following series diverges?

- (A) $\sum 1/n^2$ conv
- (B) $\sum 1/(n^2 + n)$ conv
- (C) $\sum n/(n^3 + 1)$
- (D) $\sum \frac{n}{\sqrt{4n^2 - 1}}$
- (E) none of the preceding.

$$\frac{n}{2\sqrt{4n^2 - 1}} \approx \frac{1}{4}$$

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4. For which of the following series does the Ratio Test fail?

(A) $\sum 1/n!$

(B) $\sum n/2^n$

(C) $1 + 1/2^{3/2} + 1/3^{3/2} + 1/4^{3/2} + \dots$

(D) $(\ln 2)/2^2 + (\ln 3)/2^3 + (\ln 4)/2^4 + \dots$

(E) $\sum n^n/n!$

Handwritten notes for question 4:
 $(\frac{1}{2})^{3/2}$
 $\frac{1}{3^{3/2}} \times 2^{3/2}$
 $\frac{\ln 3}{2^2} \frac{2^2}{\ln 2}$
 $\frac{1}{2} \frac{\ln 4}{\ln 3}$
 $(\frac{2}{3})^{3/2}$
 $(\frac{2}{4})^{3/2}$

5. Which of the following alternating series diverges?

(A) $\sum (-1)^{n-1}/n$

(B) $\sum (-1)^{n+1}(n-1)/(n+1)$

(C) $\sum (-1)^{n+1}/\ln(n+1)$

(D) $\sum \frac{(-1)^{n-1}}{\sqrt{n}}$

(E) $\sum (-1)^{n-1}n/n^2 + 1$

Handwritten note for question 5: $2\sqrt{2n}$

6. Which of the following series converges conditionally?

(A) $3 - 1 + 1/9 - 1/27 + \dots$

(B) $\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} - \dots$

(C) $1/2^2 - 1/3^2 + 1/4^2 - \dots$

(D) $1 - 1.1 + 1.21 - 1.332 + \dots$

(E) $1/(1*2) - 1/(2*3) + 1/(3*4) - 1/(4*5) + \dots$

Handwritten notes for question 6: $\frac{1}{3}, \frac{1}{9}, \frac{1}{3}$

7. Suppose $f(x)$ is a function with Taylor series converging to $f(x)$ for all $x \in \mathbb{R}$.

If $f(0) = 2$, $f'(0) = 2$ and $f''(0) = 3$ for $n \geq 2$ then $f(x) =$

(A) $3e^x + 2x - 1$

(B) $e^{3x} + 2x + 1$

(C) $e^{3x} - x + 1$

(D) $3e^x - x - 1$

(E) $3e^x + 5x + 5$

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$$\frac{\ln\left(\frac{1}{n^3}\right)}{\frac{1}{n^3}}$$

$$\frac{\frac{1}{n^3}}{\frac{1}{n^3}}$$

$$\frac{1}{n^3}$$

$$\frac{1}{n^3}$$

8. Which of the following series converge?

- (I) $\sum_{n=1}^{\infty} \frac{\ln(n^{-3})}{n^{-3}}$ (II) $\sum_{n=1}^{\infty} \frac{\ln 3}{3n}$ (III) $\sum_{n=1}^{\infty} \frac{n}{3^n}$

- (A) I only (B) II only (C) III only
 (D) None (E) I and III

9. What is the Taylor series for $f(x) = e^x$ about $x = 1$?

- (A) $\sum_{n=0}^{\infty} \frac{-(x-1)^n}{n!}$ (B) $\sum_{n=0}^{\infty} \frac{-e(x-1)^n}{n!}$ (C) $\sum_{n=0}^{\infty} \frac{(x-1)^n}{n!e}$
 (D) $\sum_{n=0}^{\infty} \frac{e(x-1)^n}{n!}$ (E) $\sum_{n=0}^{\infty} \frac{(x-1)^n}{n!}$

$$\frac{x^n}{n!}$$

$$\frac{e(x-1)^n}{n!}$$

10. Let $\{a_n\}$ be a sequence of positive real numbers such that

$$\frac{1}{2} \leq \frac{a_{n+1}}{a_n} \leq \frac{n+4}{2n+1} \text{ for all } n. \text{ Then } \lim_{n \rightarrow \infty} a_n =$$

$$\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} \leq \frac{1}{2}$$

$$\Rightarrow \sum a_n \text{ converges}$$

$$\Rightarrow \lim_{n \rightarrow \infty} a_n = 0$$

- (A) 0 (B) 1/2 (C) 1
 (D) 2 (E) 4

QUESTION TWO: [30 points]

Test the following series for convergence or divergence. State the test you are using, and verify that it applies. Determine whether the convergence is absolute or conditional.

(a) $\sum_{n=3}^{\infty} \frac{1}{n(\ln n)^{100}}$ \Rightarrow Converges by integral test

$$\int_3^{\infty} \frac{1}{n(\ln n)^{100}} = \frac{(\ln n)^{-99}}{-99} \Big|_3^{\infty}$$

$$= \frac{1}{99(\ln 3)^{99}} - \frac{1}{99(\ln \infty)^{99}} = \frac{1}{99(\ln 3)^{99}} + \frac{1}{\infty} \Rightarrow \text{Integral Converges}$$

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$$(b) \sum_{n=0}^{\infty} \frac{n^{2n}}{(1+2n^2)^n} \Rightarrow \text{converges by the } n\text{th root test absoli}$$

$$\lim_{n \rightarrow \infty} \sqrt[n]{\frac{n^{2n}}{(1+2n^2)^n}}$$

$$= \lim_{n \rightarrow \infty} \frac{n^2}{1+2n^2} = \boxed{\frac{1}{2}} < 1 \Rightarrow \text{converges}$$

$$(c) \sum_{n=1}^{\infty} \frac{n^3 - \sqrt{n} + 6}{n^4 - n - 3} \text{ diverges by limit comparison test}$$

$$\lim_{n \rightarrow \infty} \frac{n^3 - \sqrt{n} + 6}{n^4 - n - 3} \sim \frac{1}{n}$$

$$\lim_{n \rightarrow \infty} \frac{n^3 - \sqrt{n} + 6}{n^4 - n - 3}$$

$$= 1 \Rightarrow \text{both diverge or converge}$$

$$\frac{1}{n} \text{ diverges (power series with } p=1)$$

$$\Rightarrow \text{both diverge}$$

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QUESTION THREE: [14 points]

Consider the power series $\sum_{n=1}^{\infty} \frac{(-2)^n}{\sqrt{n}} (x+3)^n$

- (a) When $x = -4$ does this series converge or diverge?
 (b) Determine all values for which the series converges.

$$\lim_{n \rightarrow \infty} \sqrt[n]{\frac{(-2)^n}{\sqrt{n}} (x+3)^n}$$

$$= \frac{-2}{n^{\frac{1}{2n}}} (x+3) \quad \therefore |x+3| \leq \frac{1}{2} \Rightarrow R = \frac{1}{2}$$

$$-1 < x+3 < \frac{1}{2}$$

$$\underline{-4} < x < \underline{-2}$$

when $x = -4$

$$\sum \frac{(-2)^n (-1)^n}{\sqrt{n}}$$

$$\sum \frac{2^n}{\sqrt{n}}$$

$$\lim_{n \rightarrow \infty} \frac{2^n}{\sqrt{n}} \div \frac{1}{\sqrt{n}}$$

$$\lim_{n \rightarrow \infty} 2^n = \infty$$

both diverg
 by L.C.T

when $x = -2$

$$\sum \frac{(-2)^n}{\sqrt{n}}$$

Converges conditionally b A.S.T

\Rightarrow the series converges on the interval $(-4, -2]$

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QUESTION FOUR: [16 points]

Consider the integral $\int x \cos(x^3) dx$.

- (a) Write down the Maclaurin series for $\cos(x)$, $\cos(x^3)$, and $x \cos(x^3)$.
 (b) Evaluate $\int x \cos(x^3) dx$ as an infinite series.

$$\cos(x) = \frac{(-1)^n x^{2n}}{2n!}$$

$$\cos(x^3) = \frac{(-1)^n x^{6n}}{2n!}$$

Maclaurin

$$\cos(x^3) = 1 + \frac{x^6}{2!} + \frac{x^{12}}{4!} + \dots$$

$$\cos(x^3) = 1 + \frac{x^6}{2!} + \frac{x^{12}}{4!} + \frac{x^{18}}{6!} + \dots$$

$$x \cos(x^3) = x + \frac{x^7}{2!} + \frac{x^{13}}{4!} + \frac{x^{19}}{6!} + \dots$$

$$\int_0^1 x \cos(x^3) dx = x + \frac{x^7}{2!} + \frac{x^{13}}{4!} + \frac{x^{19}}{6!} + \frac{x^{25}}{8!} + \dots$$

$$\int_0^1 x \cos(x^3) dx = \frac{2x + x^7}{2} + \dots$$

$$\int_0^1 x \cos(x^3) dx = \frac{2}{3}$$

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Birzeit University
Department of Mathematics
Math 132

Key

Final exam

Name :

Instructor:

Summer/2009

Number: ...

Section :

Q#1 (72%) Circle the correct answer.

1) $\int_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} \frac{dx}{(1+x^2) \tan^{-1} x} =$

a) $\ln 4 - \ln 3$

c) $\ln 3 - \ln 2$

$\ln \frac{11}{5} x - \frac{6}{\pi}$

b) $\ln 2$

d) $\frac{\pi}{12}$

2) If $y = (\ln x)^x$ then $\frac{dy}{dx} =$

a) $(\ln x)^x \left(\frac{1}{\ln x} + \ln x \right)$

c) $x (\ln x)^{x-1}$

b) $(\ln x)^{x-1}$

d) $(\ln x)^x \left[\frac{1}{\ln x} + \ln(\ln x) \right]$

3) $\int_1^{e^3} \frac{1}{x \sqrt{1 + \ln x}} dx =$

a) $\ln \frac{4}{3}$

b) 2

c) $\frac{4}{3}$

d) $\ln \frac{3}{2}$

4) $\int_1^2 \frac{\sinh(\ln x)}{x} dx =$

a) 0

c) $\frac{1}{4}$

b) 1

d) $\frac{1}{2}$

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5) if $y = \tan^{-1}\left(\frac{1}{x}\right)$ then $\csc y =$

a) $\frac{\sqrt{1+x^2}}{x}$

b) $\sqrt{1+x^2}$

c) $\frac{x}{\sqrt{1+x^2}}$

d) $\frac{1}{\sqrt{1+x^2}}$

6) if $y = 5^{\ln x}$ then $\frac{dy}{dx}$ when $x=1$ is:

a) 0

b) $-\ln 5$

c) $\ln 5$

d) 1

7) $\int_0^1 e^{\sqrt{x}} dx =$

a) 0

b) 4

c) 3

d) 2

8) $\int x^2 e^{3x} dx =$

a) $\frac{1}{3}x^2 e^{3x} + \frac{2}{9}x e^{3x} + \frac{2}{27}e^{3x} + c$

b) $\frac{1}{3}x^2 e^{3x} + \frac{2}{9}x e^{3x} - \frac{2}{27}e^{3x} + c$

c) $\frac{1}{3}x^2 e^{3x} - \frac{2}{9}x e^{3x} + \frac{2}{27}e^{3x} + c$

d) $-\frac{1}{3}x^2 e^{3x} + \frac{2}{9}x e^{3x} - \frac{2}{27}e^{3x} + c$

9) $\int \frac{dx}{(x+2)\sqrt{x^2+4x}} =$

a) $\sqrt{x^2+4x} + c$

b) $\frac{1}{2} \sec^{-1} \left| \frac{x+2}{2} \right| + c$

c) $\frac{1}{2} \ln |x^2+2x| + c$

d) $\sinh^{-1}(x+2) + c$

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10) If $f(x) = xe^x$, then $(f^{-1})'(e) =$

- (a) $\frac{1}{e^e(e+1)}$ (b) $e^e + ee^e$ (c) $\frac{1}{2e}$ (d) None of the above

$x e^x \rightarrow e^x$

11) Consider the improper integrals:

(i) $\int_3^{\infty} \frac{dx}{(x-3)^2}$

(ii) $\int_{-5}^{\infty} \frac{dx}{\sqrt{x+5}}$

- (a) only integral (i) converge
 (b) both integrals converge
 (c) both integrals diverge
 (d) only integral (ii) converges

12. The power series $x + \frac{x^2}{2} + \frac{x^3}{3} + \dots + \frac{x^n}{n} + \dots$ converges if and only if

- (a) $-1 < x < 1$
 (b) $-1 \leq x \leq 1$
 (c) $-1 \leq x < 1$
 (d) $-1 < x \leq 1$

$\frac{x}{1-x} = 1 + x + x^2 + \dots + x^{n-1}$

$(1+x)^{\frac{1}{2}} = 1 + \frac{1}{2}x - \frac{1}{8}x^2$

13. The Maclaurin series of order 3 for $f(x) = \sqrt{x+1}$ is

- (a) $1 + \frac{x}{2} - \frac{x^2}{4} + \frac{3x^3}{8}$
 (b) $1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16}$
 (c) $1 - \frac{x}{2} + \frac{x^2}{8} - \frac{x^3}{16}$
 (d) $1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{8}$

??

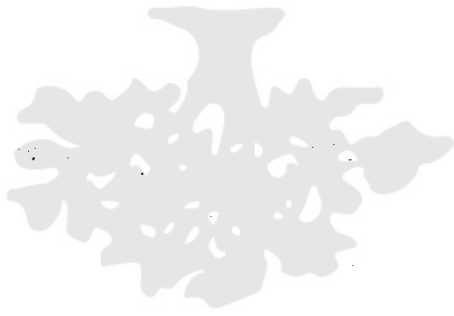
14) Determine whether $\int_2^{30} \frac{dx}{(x-3)^{2/3}}$ converges or diverges. If the integral

- converges, find its value
 a) converges, 15
 b) converges, 3
 c) converges, 1
 d) diverges.

$\int_2^{30} \frac{dx}{(x-3)^{2/3}} = \int_2^{30} (x-3)^{-2/3} dx$
 $= \left[3(x-3)^{1/3} \right]_2^{30}$
 $= 3 \cdot 27^{1/3} - 3 \cdot 1^{1/3}$
 $= 3 \cdot 3 - 3 \cdot 1$
 $= 9 - 3 = 6$

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15) If $(\frac{1+i}{1-i})^4 + z = 2+i$ then $z =$

a) $2-i$

b) $1-2i$

c) $2+4i$

d) $1+i$

16) The series $\sum_1^{\infty} \frac{n^n}{2^n 3^n}$

a) Diverges by Ratio test

b) Converges by n^{th} term test

c) Converges by Integral test

d) Converges by n^{th} root test

17) The series $\sum_1^{\infty} (-1)^n (\frac{n^2}{2n^4 + 5})$

a) Converges by limit comparison test

b) Diverges by n^{th} term test

c) Converges by n^{th} term test

d) Converges absolutely

18) The sequence $\{a_n\} = \{ \frac{1+(-1)^n}{n} \}$

a) Converges to 1

b) Converges to 0

c) Converges to 2

d) Diverges

19) $\int \sin^{-1} x \, dx =$

a) $x \sin^{-1} x - 2\sqrt{1-x^2} + c$

b) $x \sin^{-1} x + 2\sqrt{1-x^2} + c$

c) $x \sin^{-1} x - \sqrt{1-x^2} + c$

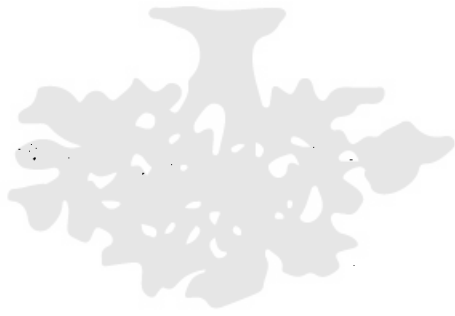
d) $x \sin^{-1} x + \sqrt{1-x^2} + c$

$u = \sin^{-1} x \quad dv = dx$
 $du = \frac{1}{\sqrt{1-x^2}} dx \quad v = x$

$x \sin^{-1} x + \frac{1}{2} \int \frac{-2x \, dx}{\sqrt{1-x^2}}$

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$$20) \int \frac{dx}{\sqrt{2x-x^2}} =$$

a) $2\sqrt{2x-x^2} + c$

c) $\sin^{-1}(x-2) + c$

b) $\sin^{-1}(x-1) + c$

d) $\sec^{-1}(x-1) + c$

21) The series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{(n+1)(n+2)}}$

a) Converge conditionally

c) Diverges

b) Converge absolutely

d) Converges to 2

22) $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$

$\frac{1}{\ln n} > \frac{1}{n}$

a) Converges Absolutely

c) Diverges by alternating series theorem

b) Converges conditionally

d) Diverges by n^{th} -term test

$e^x = e^x$

23) If $\cosh x + \sinh x = e$ then $x =$

a) e

c) $\ln 5$

b) 1

d) $\ln \sqrt{5}$

24) $\int_3^4 \frac{3dx}{x^2+x-2} =$

a) $\ln \frac{5}{4}$

b) $\ln \frac{4}{5}$

c) $\ln \frac{8}{5}$

d) $\ln \frac{5}{8}$

$\int \frac{1}{x-1} - \frac{1}{x+2}$

$\ln \frac{x-1}{x+2} \Big|_3^4 = \ln \frac{3}{5} \times \frac{5}{2}$

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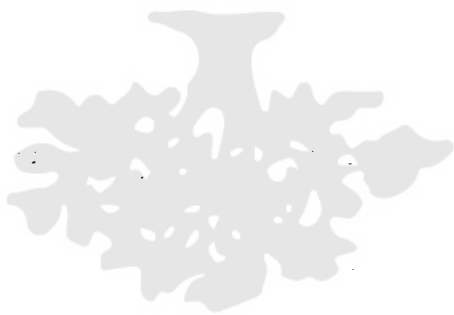
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Q2(9%) Use series to find an estimate for $\int_0^{\frac{1}{2}} \frac{\tan^{-1}x}{x} dx$ with an error of magnitude less than 10^{-3}



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Q3(10%) Solve $z^4 = -81$ in the field of complex numbers



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Q4)(10%) Sketch the graph of the conic section

$$9x^2 + 25y^2 + 18x - 100y = 116 \quad \text{indicating}$$

a) foci

b) vertices

c) eccentricity.....

d) directrices.....

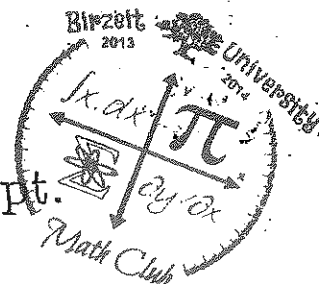
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Birzeit University
 Math. & Comp. Science Dept.
 Math. 132



Dr. Marwan Awartani

Final Exam

Fall ~~2013~~

Student Name: _____ Number: _____ Section: _____

Q1: (60 points) Circle the MOST correct answer:

1. $\int_e^{e^2} \frac{1}{x \ln x} dx =$

- (a) 1
- (b) $\ln 2$
- (c) $\ln(\frac{1}{2})$
- (d) 0

$u = \ln x$
 $du = \frac{dx}{x}$ $dx = x du$
 $= \int_e^{e^2} \frac{1}{u} du = \ln|u| \Big|_e^{e^2} = \ln|\ln x| \Big|_e^{e^2}$
 $= \ln \ln e^2 - \ln \ln e$
 $= \ln 2$

2. The curve with parametric equations $x = \sin t, y = \cos t, -\infty < t < \infty$.

- (a) a segment of a parabola
- (b) A circle
- (c) An ellipse
- (d) A hyperbola

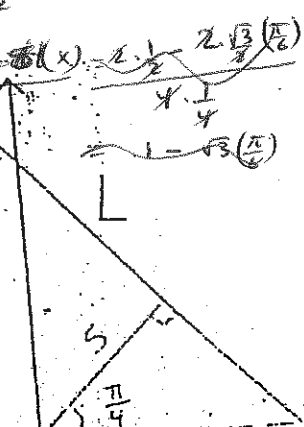
3. The slope of the curve $x = \sin 2t, y = \cos t$ at $t = +\frac{\pi}{6}$ is:

- (a) 0
- (b) 1
- (c) $\frac{1}{2}$
- (d) $-\frac{1}{2}$

$x = 2 \sin t \cos t$
 $x = 2 \sin \frac{\pi}{6} \cos \frac{\pi}{6} = 2(\frac{\sqrt{3}}{2} \cdot \frac{1}{2}) = \frac{\sqrt{3}}{2}$
 $x = 2 \sin t \cos t$
 $y = \cos t \Rightarrow y' = -\sin t = -\frac{1}{2}$

4. The polar equation of straight line l , is:

- (a) $r \cos(\theta + \frac{\pi}{4}) = -5$
- (b) $r \cos(\theta - \frac{\pi}{4}) = 5$
- (c) $r \sin(\theta + \frac{\pi}{4}) = -5$
- (d) $r \cos \theta = 5$



5. The graphs of the curves with polar coordinates $r = \sin \theta$, $r = -\cos \theta$ intersects at:

- (a) Only at the origin.
- (b) Only when $\tan \theta = -1$
- (c) At exactly two points.
- (d) At exactly three points.

6. The equation of $x^2 + 5xy + y^2 = 3$ is

- (a) Circle
- (b) Ellipse
- (c) A hyperbola
- (d) A parabola

7. One of the following is not an improper integral

(a) $\int_0^{10} \frac{\sin x}{x} dx$ $\left(x - \frac{x^3}{3!} \right)$

(b) $\int_{-\infty}^2 x dx$

(c) $\int_0^1 \frac{1}{\sqrt{x}} dx$

(d) $\int_0^{\infty} \frac{dx}{x^2 - 1}$

8. $\int \sin^2 x \cos^3 x dx =$ $\int \sin^2 x \cos^2 x \cos x dx$
 $= \int u^2 (1-u^2) du = \int (u^2 - u^4) du$
 $= \frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + C$

(a) $\frac{\sin^3 x \cos^4 x}{12} + c$

(b) $\frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + c$

(c) $\frac{\cos^3 x \sin^4 x}{15} + c$

(d) $\frac{\sin^3 x}{15} (5 + 3 \sin^2 x) + c$

9. If a particle moves on a parametric curve described by $x = t^2$, $y = \sqrt{1-t^4}$, $-1 \leq t \leq 1$, then

- (a) The initial point is (1,0) and the end point is (0,1).
- (b) the motion is clockwise.
- (c) the motion is counter clockwise.
- (d) None of the above.

10. $\lim_{x \rightarrow 0^+} (1+x)^{\frac{1}{x}}$

- (a) e
- (b) 1
- (c) $\frac{1}{e}$
- (d) 0

$u = \frac{1}{x}$
 $du = -\frac{dx}{x^2} \rightarrow dx = -x^2 du$
 $x \rightarrow 0 \text{ as } u \rightarrow \infty$

$\lim_{u \rightarrow \infty} \left(1 + \frac{1}{u}\right)^u$

10. The graph of the curves with polar coordinates $r = 1 + \cos \theta$ and $r = 1 - \cos \theta$ intersects in

- (a) One point only
- (b) Two points only
- (c) Three points only
- (d) Four points only

11. The slope of the polar curve $r = 1 + 2 \cos \theta$ at the origin is

- (a) 1
- (b) -1
- (c) $\sqrt{3}$
- (d) $\pm\sqrt{3}$

12. $\lim_{x \rightarrow 0^+} (1+x)^{\frac{1}{x}} =$

- (a) e
- (b) 1
- (c) $\frac{1}{e}$
- (d) 0

Handwritten solution for Q12:
 $u = \tan^{-1} x$
 $x = \tan u$
 $dx = \sec^2 u \cdot du$
 $\lim_{x \rightarrow 0} \frac{\tan^{-1} x}{x} = \lim_{u \rightarrow 0} \frac{u}{\tan u}$
 $= \lim_{u \rightarrow 0} \frac{u \cos u}{\sin u}$

13. $\lim_{x \rightarrow 0} \frac{\tan^{-1} x}{x} =$

- (a) 1
- (b) -1
- (c) 0
- (d) ∞

Handwritten solution for Q13:
 $u = \tan^{-1} x$
 $du = \frac{dx}{1+x^2}$
 $dx = (1+x^2) \cdot du$
 $= \lim_{x \rightarrow 0} \frac{(1+x^2)}{x} \cdot dx = \frac{1 + \tan^2 u}{\tan u} \cdot dx$
 $= \int \frac{\sec^2 u}{\tan u} \cdot dx$

14. $7^{7^7} =$

- (a) $\frac{1}{5}$
- (b) $\frac{1}{5}$
- (c) 5
- (d) 7

Handwritten solution for Q14:
 $= \int \frac{1}{\cos^2 u} \cdot \frac{\cos u}{\sin u} \cdot dx = \int \frac{1}{\sin u \cos u}$

15. The eccentricity of the conic section $\frac{x^2}{9} - \frac{y^2}{16} = 1$ is

- (a) $\frac{5}{3}$
- (b) $\frac{4}{5}$
- (c) $\frac{3}{4}$
- (d) $\frac{4}{3}$

Handwritten solution for Q15:
 $a = 3$ $b = 4$
 $c^2 = a^2 + b^2$
 $= 9 + 16$
 $c = 5$
 $e = \frac{c}{a} = \frac{5}{3}$

16. The length of the polar curve $r = 4 \cos \theta$, $0 \leq \theta \leq \frac{\pi}{2}$ is

- (a) π
- (b) 2π
- (c) 3π

17. The surface area generated by revolving $r = 4 \cos \theta$ $0 \leq \theta \leq \frac{\pi}{2}$ about x-axis is

- (a) 2π
- (b) 4π
- (c) 8π
- (d) 16π

18. The Cartesian coordinates of the point $P(-4, \frac{\pi}{4})$ is

- (a) $(-2\sqrt{2}, 2\sqrt{2})$
- (b) $(-2\sqrt{2}, -2\sqrt{2})$
- (c) $(2\sqrt{2}, 2\sqrt{2})$
- (d) $(2\sqrt{2}, -2\sqrt{2})$

19. The angle θ that eliminate xy term in $2x^2 + \sqrt{3}xy + y^2 - 2y = 6$ is

- (a) $\frac{\pi}{3}$
- (b) $\frac{\pi}{4}$
- (c) $\frac{\pi}{6}$
- (d) $\frac{\pi}{2}$

20. $\sec^{-1}(-\sqrt{2}) = -\sqrt{2} = \sec y$

21. $\frac{\pi}{4} \quad -\sqrt{2} = \frac{1}{\cos y}$

22. $\frac{-\pi}{4} \quad \cos y = \frac{-1}{\sqrt{2}}$

23. $\frac{3\pi}{4} \quad y = \frac{\pi}{4} + \frac{\pi}{4} = \frac{5\pi}{4}$

(24) $\frac{5\pi}{4}$

$\frac{\pi}{4} - \frac{\pi}{4} = -\frac{3\pi}{4}$

II (15 points): Evaluate the following integrals

$\int \sin^{\frac{5}{2}} x$

محيطس الطلبة

2017 2016

BIRZeit UNIVERSITY



Birzeit University
Math. & Comp. Science Dept.
Math. 132

Dr. Hasan Yousef

Final Exam

Summer ~~2016~~

Student Name: _____ Number: _____ Section: _____

I (40points) : Circle the MOST correct answer:

1. If $\sinh x = \frac{-3}{4}$ then $\cosh x =$

- (a) $\frac{3}{4}$
- (b) $\frac{4}{3}$
- (c) $\frac{-3}{5}$
- (d) $\frac{-5}{4}$



2. The conic section with Foci $(\pm 1, 0)$ and vertices $(\pm 2, 0)$ is an

- (a) Ellipse
- (b) Parabola
- (c) Hyperbola
- (d) A circle

$c=1 \quad a=2$
 $a > c$
 $e > 1$

3. The conic section with eccentricity $\frac{1}{2}$ and directrix $x = 2$ has equation

- (a) $2x^2 + y^2 = 1$
- (b) $x^2 - y^2 = 1$
- (c) $y^2 - x^2 = 2$
- (d) $x^2 + \frac{4}{3}y^2 = 1$

$e = \frac{c}{a} = \frac{1}{2} \quad 2 \cdot \frac{1}{2} = 1 \quad \frac{a}{e} = 2$
 $\frac{a}{2} = \frac{c}{a} = \frac{1}{2} \quad a = 2e$
 $a^2 = 2c \quad a=1 \quad 2c=a$
 $a=1$

4. The directrix of the parabola $x = \frac{y^2}{2}$ is given by

- (a) $x = 1$
- (b) $y = \frac{1}{2}$
- (c) $y = \frac{1}{2}$
- (d) $x = \frac{1}{2}$

$\frac{x^2}{1} + \frac{y^2}{3} = 1$
 $xy = 2x$
 $4px = 2x$
 $p = \frac{1}{2}$

$a = 2e$
 $= 2 \cdot \frac{1}{2}$
 $= 1$
 $e = \frac{c}{a} = \frac{1}{2} \cdot 1$
 $c = \frac{1}{2}$

~~RAE~~

$D = \frac{a}{e} = 2$

$a = 2e$

$e = \frac{c}{a} = \frac{1}{2}$

$a = 1$

$a = 2c \rightarrow c = \frac{1}{2}$

$b^2 = a^2 - c^2$

$= 1 - \frac{1}{4}$
 $b = \frac{\sqrt{3}}{2}$

5. The conic section $x^2 + 4xy + \sqrt{2}y^2 + 5 = 0$ is

- (a) Ellips
- (b) Parabola
- (c) Hyperbola
- (d) A Circle

$$x^2 + \sqrt{2}y^2 = -4xy - 5$$

$$\frac{x^2 + \sqrt{2}y^2}{-4xy - 5} = \frac{-4xy}{-4xy - 5}$$

$$= \frac{x}{-4y} + \frac{\sqrt{2}y}{-4x} = \frac{5}{-4xy}$$

6. $x = \cos 2t, y = \sin^2 t, 0 \leq t \leq \frac{\pi}{4}$ represents

- (a) half of a circle
- (b) half of an Ellipse
- (c) a line segment
- (d) a parabola

$$-4x^2 - 4\sqrt{2}y^2 = -5$$

7. The slope of the Ellipse $x = 2 \sin t, y = 3 \cos t, 0 \leq t \leq 2\pi$ at the point $(\sqrt{2}, \frac{3}{\sqrt{2}})$ is

- (a) $-\frac{2}{3}$
- (b) 3
- (c) $\frac{3}{2}$
- (d) $\frac{3}{2}$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{4 \sin^2 t}{a^2} + \frac{9 \cos^2 t}{b^2} = 1$$

$$t = \sin^{-1}\left(\frac{x}{2}\right) = \frac{\pi}{4}$$

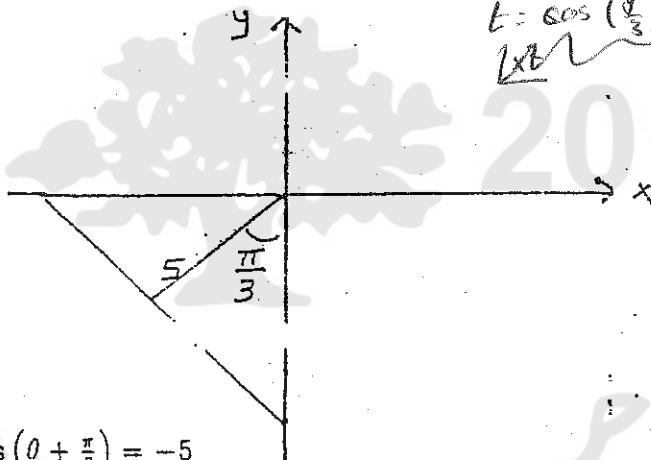
$$t = \cos^{-1}\left(\frac{y}{3}\right) = \frac{\pi}{4}$$

$$\frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2}{2\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}}$$

$$\frac{3}{3\sqrt{2}} = \frac{1}{\sqrt{2}}$$

8. An equation of the line in the figure is



- (a) $r \cos\left(\theta + \frac{\pi}{6}\right) = -5$
- (b) $r \sin\left(\theta + \frac{2\pi}{3}\right) = -5$
- (c) $r \cos\left(\theta + \frac{5\pi}{6}\right) = 5$
- (d) $r \sin\left(\theta + \frac{\pi}{6}\right) = -5$

9. The polar Equation of the circle with center $P(-2, \frac{\pi}{4})$ and radius 4 is

- (a) $r = -4 \sin\left(\theta + \frac{\pi}{4}\right)$
- (b) $r = -4 \sin\left(\theta + \frac{3\pi}{4}\right)$
- (c) $r = 4 \sin\left(\theta - \frac{3\pi}{4}\right)$
- (d) $r = 4 \sin\left(\theta - \frac{\pi}{4}\right)$

$$\int \frac{x^2 \cdot dx}{2x\sqrt{1+x^2}}$$

$$= \int \frac{x \cdot dx}{2\sqrt{1+x^2}} = \int \frac{\sqrt{u} \cdot du}{2\sqrt{1+u}}$$

$$u = x^2$$

$$du = 2x \cdot dx$$

$$dx = \frac{du}{2x}$$

Q2: (15 points) (a) $\int \frac{x^2 dx}{\sqrt{1+x^2}} =$

$$u = 1+x^2$$

$$du = 2x \cdot dx$$

$$dx = \frac{du}{2x}$$

$$= \int \frac{x^2 \cdot dx}{2x\sqrt{u}} = \int \frac{\sqrt{u-1} \cdot du}{2\sqrt{u}}$$

$$= \frac{1}{2} \int \sqrt{\frac{u-1}{u}} \cdot du = \frac{1}{2} \int \sqrt{1 - \frac{1}{u}} \cdot du$$

$$= \int \frac{x^2 \cdot \sqrt{1+x^2} \cdot du}{x(u)} = \int \sqrt{u^2-1} \cdot du$$

$$= \int (u^2-1)^{\frac{1}{2}} \cdot du$$

$$u = \sqrt{1+x^2}$$

$$du = \frac{2x \cdot dx}{2\sqrt{1+x^2}}$$

$$du = \frac{x \cdot dx}{\sqrt{1+x^2}}$$

$$dx = \frac{\sqrt{1+x^2} \cdot du}{x}$$

(b) $\int \frac{x^2+x+1}{x^3+x} dx =$

$$u = \sqrt{1+x^2}$$

$$u^2 = 1+x^2$$

$$x = \sqrt{u^2-1}$$

$$\frac{x^2+x+1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$$

(c) $\lim_{x \rightarrow \infty} \left(\frac{x-1}{x-2} \right)^x = \lim_{x \rightarrow \infty} \frac{(x-1)^x}{(x-2)^x} = \lim_{x \rightarrow \infty} \frac{x(x-1)^{x-1}}{x(x-2)^{x-1}}$

$$= \lim_{u \rightarrow \infty} \frac{(u)^{u+1}}{(u)^{u+2}}$$

$$= \lim_{u \rightarrow \infty} u^{(u+1)-(u+2)}$$

$$= \lim_{u \rightarrow \infty} u^{-1}$$

$$= \lim_{u \rightarrow \infty} \frac{1}{u} = 0$$

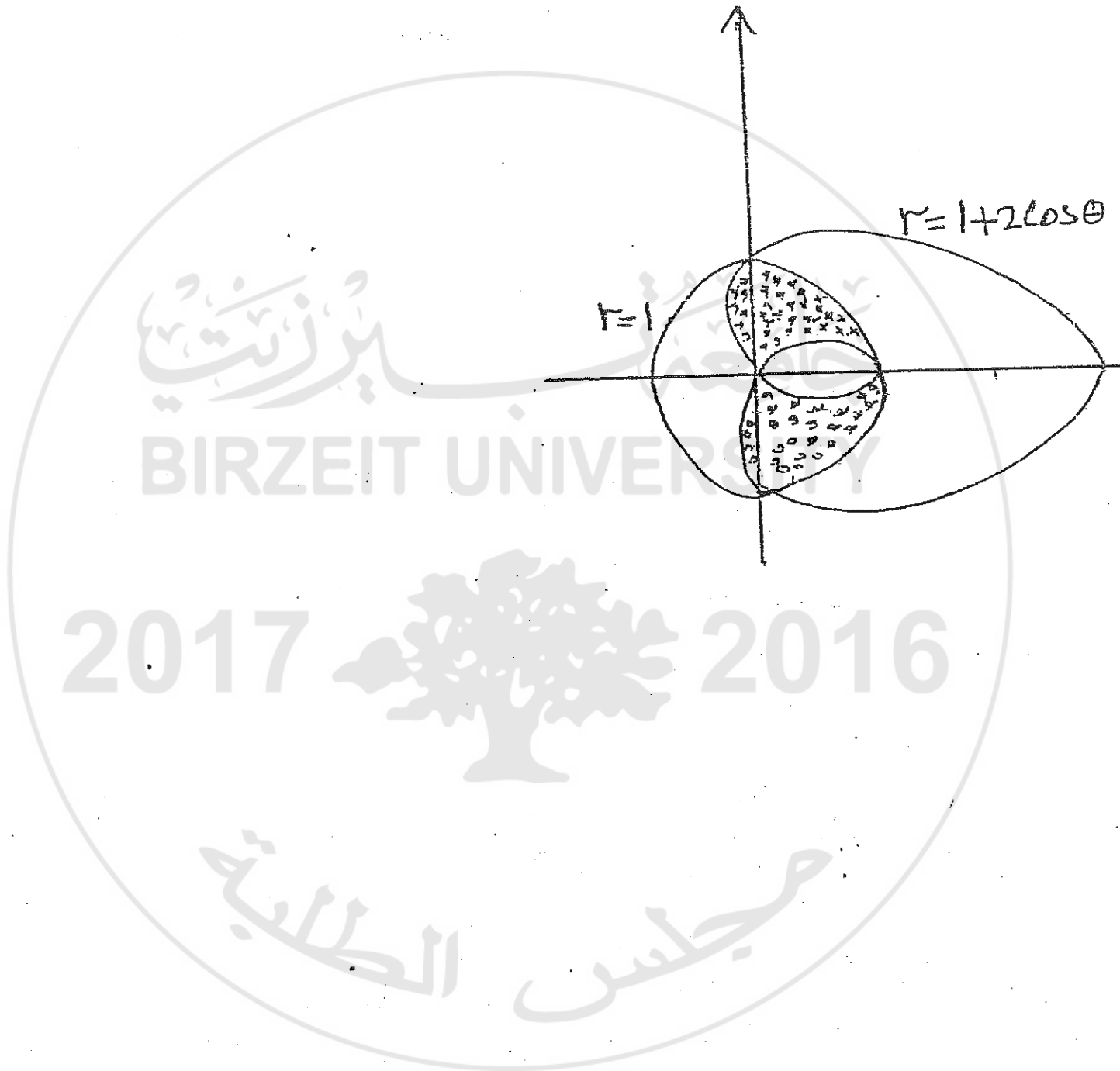
$$u = x-1$$

$$du = dx$$

$$x = u+1$$

$$x \rightarrow \infty \quad u \rightarrow \infty$$

IV (15 points): Find the area of the shaded region.





Birzeit University
Math. & Comp. Science Dept.
Math. 132

Final Exam

Second Semester

The Name of Discussion Teacher:

Student Name: _____ Number: _____ Section: _____

I (50 points) : Circle the MOST correct answer:

1. If $\sinh x = 3$ then $\cosh(-x) =$

- (a) $\sqrt{10}$
- (b) $-\sqrt{10}$
- (c) $\sqrt{8}$
- (d) $\sqrt{10}, -\sqrt{10}$



2. $\int \frac{ax+b}{x^3+x^2} dx$ is a rational function if

- (a) $b = 0$
- (b) $a = b$
- (c) $a = -b$
- (d) $a = -b$

3. If $2^{2^x} = 4 \cdot 2^x$ and $x > 0$ then $x =$

- (a) 2
- (b) -1
- (c) 2, -1
- (d) 4

$$2 \ln 2^{2^x} = x \ln 4 \cdot 2$$

$$x = \frac{\ln 4 \cdot 2}{\ln 2} = \ln 2 \cdot 2$$

$$u = e^x$$

$$du = e^x \cdot dx$$

$$x=1 \rightarrow u=e$$

$$x=0 \rightarrow u=1$$

4. $\int_0^1 \frac{e^x - e^{-x}}{e^x + e^{-x}} dx =$

- (a) $\tanh^{-1}(1)$
- (b) $\ln(e^2 + 1) - \ln 2$
- (c) $\ln(e^2 + 1) - \ln 2 - 1$
- (d) None of the above

$$= \int_1^e \frac{u - \frac{1}{u}}{u + \frac{1}{u}} \cdot \frac{1}{u} du = \int_1^e \frac{u^2 - 1}{u(u^2 + 1)} \cdot \frac{1}{u} du$$

$$= \int_1^e \frac{u^2 - 1}{u^2(u^2 + 1)} du = \int_1^e \left(\frac{1}{u} - \frac{2}{u^2 + 1} \right) du$$

Handwritten notes:

$\frac{u^2-1}{u(u^2+1)} = \frac{A}{u} + \frac{B}{u^2+1}$

$u^2-1 = A(u^2+1) + B(u)$

$A=1$

$u=1$

$2A+B=0$

$B=-2A$

$B=-2$

2017 2016

$$= 2x \ln x - \frac{2(u e^u - e^u)}{\ln 3}$$

$$f(x) = u \quad g(x) = e^u$$

$$f'(x) = 1 \quad \int g(x) = e^u$$

$$\int \int g(x) = e^u$$

$$= \int \frac{\ln x^2}{\ln 3} dx = \frac{1}{\ln 3} \int \ln x^2 dx$$

$$5. \int \log_3 x^2 dx = \frac{1}{\ln 3} \int u \cdot \frac{x}{2} dx \quad u = \ln x^2$$

$$du = \frac{2x}{x^2} dx$$

(a) $2x \log_3 x^2 - x + c$

(b) $2x \log_3 x - x + c$

(c) $\frac{2}{\ln 3} (x \ln x + x) + c$

(d) $\frac{2}{\ln 3} (x \ln x - x) + c$

$$\frac{1}{2 \ln 3} \int u \cdot e^u du$$

$$z = e^u$$

$$dv = u du$$

$$x = \frac{e^u}{2}$$

$$dv = \frac{x}{2} du$$

$$dz = e^u du$$

$$v = \frac{u^2}{2}$$

$$\frac{u}{2} = \ln x$$

6. If $\int_0^b f(x) dx$ diverges and $\int_a^b g(x) dx$ diverges then $\int_a^b f(x)g(x) dx$

(a) converges always

(b) diverges always

(c) Can't decide

(d) Converges if $f(x) = g(x)$

$$= \frac{e^u u^2}{2} - \int \frac{u^2}{2} e^u du$$

7. A line $y = ax + b$ and a curve of a function $y = e^x$ can intersect in at most

(a) 1 point

(b) 2 points

(c) 3 points

(d) 4 points

$$ax + b = e^x$$



8. If $\sin^{-1} x = \frac{\pi}{3}$ then $\cos^{-1} x =$

(a) $\frac{2\pi}{3}$

(b) $\frac{5\pi}{6}$

(c) $\frac{\pi}{4}$

(d) $\frac{\pi}{3}$

$$x = \frac{\sqrt{3}}{2}$$

$$x = \sin u$$

$$u = \sin^{-1} x$$

$$du = \frac{dx}{\sqrt{1-x^2}}$$

$$dx = \sqrt{1-x^2} du$$

$$= \cos u du$$

$$\frac{x=1}{u=\pi/4} \quad \frac{x=0}{u=0}$$

$$9. \int_0^{\pi/2} \sin x dx + \int_0^{\pi/2} \sin^{-1} x dx = \left[-\cos x \right]_0^{\pi/2} + \left[\sin u \right]_0^{\pi/4}$$

(a) π

(b) $\frac{\pi}{2}$

(c) 1

(d) $\frac{1}{2}$

$$= 0 + 1 + \frac{1}{\sqrt{2}} - 0$$

$$\left(\frac{2+1}{\sqrt{2}} \right) \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2+\sqrt{2}}{2}$$

10. One of the following is not an improper integral.

(a) $\int_0^{10} \frac{\sin x}{x} dx$

(b) $\int_{-\infty}^a x dx$

(c) $\int_0^1 \frac{1}{\sqrt{x}} dx$

(d) $\int_{-2}^{\infty} \frac{dx}{x-1}$

11. $\int \sin^2 x \cos^3 x dx =$

(a) $\frac{\sin^3 x \cos^4 x}{12} + c$

(b) $\frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + c$

(c) $\frac{\cos^3 x \sin^4 x}{12} + c$

(d) $\frac{\sin^3 x}{15} (5 + 3 \sin^2 x) + c$

12. The function $f(x) = \ln|x + 2|$ has domain

(a) $(-2, \infty)$

(b) $(0, \infty)$

(c) $[-1, \infty)$

(d) $[0, \infty)$

13. $\int x^3 e^x dx =$

(a) $\frac{x^4}{4} e^x + c$

(b) $\frac{x^4}{4} e^x - e^x + c$

(c) $e^x(x^3 - 3x^2 + 6x - 6) + c$

(d) $e^x(-x^3 + 3x^2 - 6x + 6) + c$

14. If a particle moves on a parametric curve described by $x = t^2, y = \sqrt{1 - t^2}$, $t \leq 1$, then

(a) The initial point is $(1, 0)$ and the end point is $(0, 1)$.

(b) The motion is clockwise.

(c) The motion is counter clockwise.

15. The total distance travelled by a particle moving on the curve $r = 2 \sin \theta$ from $\theta = 0$ to $\theta = 2\pi$ is

(a) 4π

(b) 2π

(c) π

(d) 8π

16. The slope of the tangent lines to the curve $r = 1 - 2 \cos \theta$ at the origin are

(a) 0

(b) $\pm\sqrt{3}$

(c) ± 1

(d) not defined

$$y = \pm \frac{b}{a} x$$

$$a = 4 \quad c = 5 \\ b = 3$$

$$e = \frac{c}{a} = \frac{5}{4}$$

$$x = \frac{a}{e}$$

17. The directrices of the hyperbola $\frac{(x+2)^2}{16} - \frac{(y-1)^2}{9} = 1$ are

- (a) $y = \pm \frac{16}{5}$
- (b) $x = \pm \frac{16}{5}$
- (c) $x = \frac{-26}{5}, x = \frac{6}{5}$
- (d) $y = \frac{-26}{5}, y = \frac{6}{5}$

$$D = \frac{a}{e} = \frac{4}{\frac{5}{4}} = \frac{16}{5}$$

$$\frac{4}{\frac{5}{4}}$$

18. The polar curves $r = \cos 2\theta, r = \frac{1}{2}$ intersects in

- (a) 1 point
- (b) 2 points
- (c) 4 points
- (d) 8 points

$$x = \pm \frac{16}{5} \\ x = \frac{+16}{5} - 2 \quad x = \frac{-16}{5} - 2$$

19. The circles $x^2 + y^2 = 1$ and $(x-1)^2 + y^2 = a^2$ intersect in two point if:

- (a) $a = 2$
- (b) $2 < a$
- (c) $a < 2$
- (d) $0 < a < 2$

$$x \rightarrow 0 \\ u \rightarrow \infty$$

20. $\lim_{x \rightarrow 0^+} \frac{\ln x}{\sqrt{x}} =$

- (a) 0
- (b) $-\infty$
- (c) ∞
- (d) 1

$$u = \ln x \\ du = \frac{dx}{x} \\ \frac{dx}{x} = x \ln x \\ x = e^4 \quad u = \sqrt{x} \\ = \frac{1}{x} = \frac{2\sqrt{x}}{x} = \infty$$

77
e0

II (15 points): 1. $\lim_{x \rightarrow \infty} \left(\frac{x-1}{x-2}\right)^x$

$$= \lim_{x \rightarrow \infty} \frac{(x-1)^x}{(x-2)^x} = \frac{x \ln(x-1)}{x \ln(x-2)} = \frac{x - \frac{x^2}{2} + \frac{x^3}{3} \dots}{\ln 2 - \frac{x}{2} - \frac{x^2}{4} \dots}$$

$$f(x) = \ln(x-2) \quad f(0) = \ln 2 \\ f'(x) = \frac{1}{x-2} \quad f'(0) = -\frac{1}{2} = \ln 2 \\ f''(x) = \frac{-1}{(x-2)^2} \quad f''(0) = -\frac{1}{4}$$

2. Test for convergence $\int_{-\infty}^{\infty} \frac{dx}{e^x + e^{-x}}$.

$$= \int_{-\infty}^{\infty} \frac{dy}{u(u+\frac{1}{u})} = \int_{-\infty}^{\infty} \frac{du}{u(u^2+1)}$$

$$= \int_{-\infty}^{\infty} \left[\tan^{-1} u \right]_{-\infty}^0 + \left[\tan^{-1} \frac{1}{u} \right]_0^{\infty}$$

$$= \left[\tan^{-1}(e^x) \right]_{-\infty}^0 + \left[\tan^{-1}(e^{-x}) \right]_0^{\infty}$$

$$= \frac{\pi}{4} - \frac{\pi}{4} + \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

$$u = e^x$$

$$du = e^x \cdot dx$$

$$dx = \frac{du}{e^x} = \frac{du}{u}$$

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3. Evaluate $\int \frac{\sqrt{9-x^2}}{x} dx$.

~~$$\int \frac{\sqrt{u}}{2x^2} dy$$~~

~~$$\int \frac{\sqrt{u}}{2(9-u)} du$$~~

~~$$\int \frac{\sqrt{9-u}}{2u} du$$~~

~~$$u = 9/x^2$$~~
~~$$du = -2/x \cdot dx$$~~
~~$$dx = \frac{du}{-2/x}$$~~

$$u = x^2$$

$$du = 2x \cdot dx$$

$$dx = \frac{du}{2x}$$

~~$$a^2 = x^2$$~~

$$x = a \sin \theta$$

$$a^2 - x^2 = a^2 - a^2 \sin^2 \theta = a^2 \cos^2 \theta$$

$$dx = a \cos \theta \cdot d\theta$$

$$= \int \frac{\sqrt{a^2 - a^2 \sin^2 \theta}}{a \sin \theta} \cdot a \cos \theta \cdot d\theta$$

$$= \int \frac{\cos^2 \theta \cdot d\theta}{\sin \theta} = \int \cot \theta \csc \theta \cdot d\theta = \csc \theta + C$$

III. (20 points): (a) Sketch the graph of $r = \frac{8}{2 - \sin \theta}$ and indicate the center, vertices, foci and directrices.

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(b) Use part (a) to sketch the graph of $r = \frac{8}{2 - \sin(\theta + \frac{\pi}{4})}$

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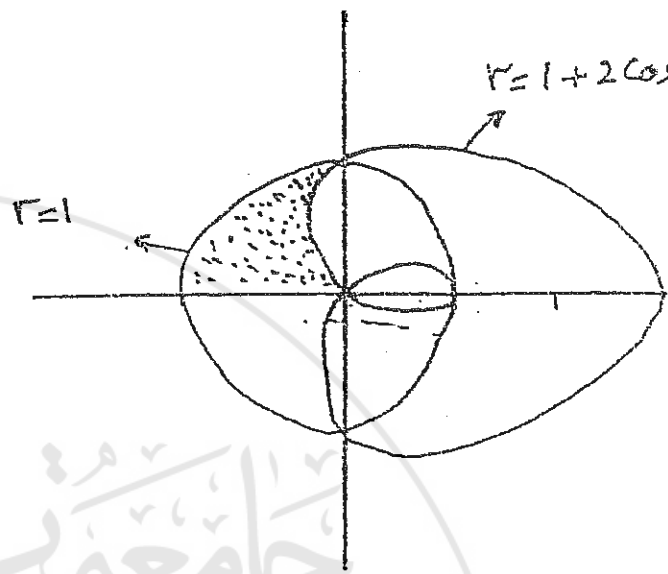
IV (20 points): (a) Sketch the graph of the conic section $r = \frac{8}{2 - 4 \cos \theta}$ indicating vertices foci and directrices in polar coordinates.

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(b) Use (a) to sketch the graph of $r = \frac{8}{2 - 4 \cos (\theta + \frac{\pi}{4})}$ indicating vertices, foci and directrices in polar coordinates.

V (15 points): Find the shaded area.



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IV (20 points): (a) Sketch the graph of the conic section $r = \frac{8}{2 - 4 \cos \theta}$ indicating vertices foci and directrices in polar coordinates.

Birzeit University- Mathematics Department
Math 132

Dr. Marwan Alogqili (Sec.2) and Dr. Marwan Awartani (Sec.1&3)

Third Exam

Fall 2002/2003

Name: Ali N. N. N. Tagatga

Number: C.I.1.2.73

There are 10 (T/F) questions and 14 multiple choice. Calculators are not allowed.

Question 1 Circle the most correct answer:

$\lim_{n \rightarrow \infty} \frac{n}{n+10^6} = \frac{1}{1} = 1$

70

1. $\sum_{n=1}^{\infty} \frac{1}{n+(10)^n}$

- (a) Converges to 0.
- (b) Converges by nth term test.
- (c) Diverges.
- (d) None.

2. One of the following p-series converges:

- (a) $\sum_{n=1}^{\infty} \frac{1}{n^{1.1}}$
- (b) $\sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$
- (c) $\sum_{n=1}^{\infty} \frac{1}{n}$
- (d) $\sum_{n=1}^{\infty} \frac{1}{n^{1/4}}$

$p > 1$

3. The sequence $a_n = (1 - \frac{1}{n^2})^n$

- (a) Diverges.
- (b) Converges to -1.
- (c) Converges to 1.
- (d) None of the above.

$\lim_{n \rightarrow \infty} (1 - \frac{1}{n^2})^n$
 $= \lim_{n \rightarrow \infty} (1 - \frac{1}{n^2})^{n \cdot \frac{1}{n^2} \cdot n^2}$
 $= \lim_{n \rightarrow \infty} (1 - \frac{1}{n^2})^{n^2 \cdot \frac{1}{n^2}}$
 $= \lim_{n \rightarrow \infty} (1 - \frac{1}{n^2})^{1/n^2}$
 $= e^{-\lim_{n \rightarrow \infty} \frac{1}{n^2}} = e^0 = 1$

$\lim_{n \rightarrow \infty} (1 - \frac{1}{n^2})^n$

$\ln f(x) = \ln(1 - \frac{1}{n^2})$
 $\lim_{n \rightarrow \infty} \ln f(x) = \lim_{n \rightarrow \infty} \frac{\ln(1 - \frac{1}{n^2})}{1/n^2}$
 $= \lim_{n \rightarrow \infty} \frac{-1/n^2}{-2/n^3} = \lim_{n \rightarrow \infty} \frac{1}{2n} = 0$

4. $\sum_{n=1}^{\infty} \tan^{-1}(n+1) - \tan^{-1}n$

- (a) $\pi/2$.
- (b) $\pi/4$.
- (c) $-\pi/4$.
- (d) None of the above.

$\tan^{-1}(2) - \tan^{-1}(1) + \tan^{-1}(3) - \tan^{-1}(2) + \tan^{-1}(4) - \tan^{-1}(3) + \dots$

$\lim_{n \rightarrow \infty} \frac{-1}{(1 - \frac{1}{n^2})n^3} = \lim_{n \rightarrow \infty} \frac{-1/n}{1 - \frac{1}{n^2}} = 0$

5. $\sum_{n=2}^{\infty} \frac{\ln n}{n}$

- (a) Diverges by integral test.
- (b) Diverging by directly comparing it with $\sum \frac{1}{n}$.
- (c) Diverging by limit comparison test with $\sum \frac{1}{n}$.
- (d) All of the above.

$\lim_{n \rightarrow \infty} \ln f(x) = 0$
 $\Rightarrow \ln f(x) = 1$
 $\Rightarrow \lim_{n \rightarrow \infty} f(x) = e^1 = e$

$\frac{\ln n}{n} > \frac{1}{n}$
 $\lim_{n \rightarrow \infty} \frac{\ln n}{n} = 0$

$\int \frac{\ln n}{n} = \int \frac{u}{x} x du$
 $= \int \frac{u^2}{2} = \frac{1}{2} u^2$

$(\tan^{-1} 2 - \tan^{-1} 1) + (\tan^{-1} 3 - \tan^{-1} 2) + (\tan^{-1} 4 - \tan^{-1} 3) + \dots + (\tan^{-1} n - \tan^{-1} (n-1))$
 $= \tan^{-1} n - \tan^{-1} 1 = \tan^{-1} n - \frac{\pi}{4}$

$$\frac{1 + \cos^2 n}{n^2} < \frac{2}{n^2}$$

6. $\sum_{n=1}^{\infty} \frac{1 + (\cos n)^2}{n^2}$

- (a) Converges.
- (b) Diverges.
- (c) Diverges by the nth term test.
- (d) Cannot determine.

7. $\sum_{n=1}^{\infty} e^{-n} = \frac{1}{e} + \frac{1}{e^2} + \frac{1}{e^3} + \dots$

- (a) $\frac{c}{c-1}$
- (b) $\frac{1}{c-1}$
- (c) $\frac{1}{1-c}$
- (d) None of the above.

$\frac{1}{e^n} = \left(\frac{1}{e}\right)^n$
 $= \frac{1}{1 - \frac{1}{e}} = \frac{1}{\frac{e-1}{e}} = \frac{e}{e-1}$ as $\frac{1}{e} = \frac{1}{e}$

$\frac{1/e}{e(1 - 1/e)} = \frac{1}{e-1}$

8. One of the following is true:

- (a) $\sum_{n=1}^{\infty} \frac{3^n}{n^{3/2}}$ diverges by ratio test.
- (b) $\sum_{n=1}^{\infty} \frac{1}{n+1}$ converges by nth root test. \times
- (c) $\sum_{n=1}^{\infty} 2^n$ converges. \times
- (d) $\sum_{n=1}^{\infty} \frac{1}{n^n}$ is a geometric series. \times

$\lim \frac{3^{n+1}}{(n+1)^{3/2}} \cdot \frac{n^{3/2}}{3^n}$
 $= \lim \frac{n^3}{(n+1)^3} = \frac{3}{2}$

9. The nth partial sum of the series $\sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}}\right)$ is

- (a) $s_n = 1 - \frac{1}{\sqrt{2}}$
- (b) $s_n = \frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}}$
- (c) $s_n = 1 - \frac{1}{\sqrt{n+1}}$
- (d) None of the above.

$\frac{1}{\sqrt{1}} - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}} + \dots + \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}}\right)$
 $= 1 - \frac{1}{\sqrt{n+1}}$

10. If $a_1 = 2, a_{n+1} = \frac{2}{n} a_n$ then $\sum_{n=2}^{\infty} a_n$

- (a) Converges by ratio test.
- (b) Diverges.
- (c) Converges by integral test.
- (d) None of the above.

$2 + 2(2) + 1(4) + \dots$

$\lim \frac{a_{n+1}}{a_n} = \frac{\frac{2}{n} a_n}{a_n} = \frac{2}{n} \rightarrow 0$

11. The series $\sum_{n=0}^{\infty} (\ln x)^n$ converges if

- (a) $-1 < x < 1$
- (b) $-e < x < e$
- (c) $e^{-1} < x < e$
- (d) None.

$\sum_{n=1}^{\infty} (\ln x)^{n-1}$
 $a_1 = (\ln x)^0 = 1$
 $a_2 = (\ln x)$
 $a_3 = (\ln x)^2 \Rightarrow r = (\ln x)$

$s_n = \frac{1}{1 - (\ln x)}$

$e^{-1} < \ln x < 1$

12. One of the following series converges:

- (a) $\sum_{n=1}^{\infty} n^2$.
- (b) $\sum_{n=1}^{\infty} \frac{n+1}{n}$.
- (c) $\sum_{n=1}^{\infty} \frac{1}{2^n}$.
- (d) $\sum_{n=1}^{\infty} \ln(1/n)$.

- 5 8

13. $\sum_{n=1}^{\infty} \frac{-5}{n}$

- (a) Converges to 0.
- (b) Converges to 1.
- (c) Diverges.
- (d) None of the above.

14. $\sum_{n=1}^{\infty} \frac{2^n}{n^2}$

- (a) Converges.
- (b) Diverges.
- (c) Converges to 0.
- (d) None.

$$\lim_{n \rightarrow \infty} \frac{2^{n+1}}{(n+1)^2} = \lim_{n \rightarrow \infty} \frac{2 \cdot 2^n}{n^2 + 2n + 1} = 2 > 1 \Rightarrow \text{div.}$$

$$\frac{(-1)^{n+2}}{(-1)^{n+1}} = (-1)^{n+2-n-1} = (-1)^1 = -1$$

Question 2. Answer by True or False:

1. The series $\sum_{n=1}^{\infty} (-1)^{n+1}$ diverges.
2. If $\lim_{n \rightarrow \infty} (a_n)^{1/n} = 1$ then $\sum_{n=1}^{\infty} a_n$ converges.
3. Any increasing sequence and bounded from above converges.
4. $\lim_{n \rightarrow \infty} \frac{x^n}{n!} = 0$.
5. The sequence $a_n = 1 + (-1)^n$ diverges.
6. If $\lim_{n \rightarrow \infty} a_n = 0$ then $\sum_{n=1}^{\infty} a_n$ converges.
7. The series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is called the harmonic series.
8. The sequence $a_n = \frac{3n+1}{n+1}$ is nondecreasing.
9. The series $\sum_{n=1}^{\infty} (\sin x)^n$ converges for any value of x .
10. If $\sum_{n=1}^{\infty} a_n$ converges then $\sum_{n=1}^{\infty} k a_n$, k is any constant, converges.

- (true) ✓
- (false) ✓
- (true) ✓
- (true) ✓
- (true) ✓
- (false) ✓
- (false) ✓
- (true) ✓
- (false) ✓
- (true) ✓

Handwritten work for question 10:

$$\lim_{n \rightarrow \infty} \frac{3(n+1) - (3n)}{(n+1)^2} = \lim_{n \rightarrow \infty} \frac{3n+3-3n}{(n+1)^2} = \lim_{n \rightarrow \infty} \frac{3}{(n+1)^2} = 0$$

0, 1, 0, 1

$$\sum_{n=2}^{\infty} (-1)^n = \frac{(-1)^2}{1-(-1)} = \frac{1}{2}$$



Birzeit University
Math. & Comp. Science Dept.
Math: 132

Third Hour Exam

Second Semester ~~XXXXXX~~

The Name of Discussion Teacher:

Student Name: _____ Number: _____ Section: _____

I (40 points): Circle the MOST correct answer:

1. The directrix of the parabola $y = 2x^2 + 4x + 1$ is

(a) $y = -\frac{1}{8}$

(b) $y = \frac{7}{8}$

(c) $y = -\frac{9}{8}$

(d) $y = \frac{9}{8}$

2. A circle of radius a and an ellipse of major semiaxis a both centered at the origin meets in

(a) 1 points

(b) 1 points

(c) 2 point

(d) 0 point

3. The quadratic equation $x^2 + 2xy + y^2 - 2x + 2y + 10 = 0$ represents

(a) Parabola

(b) Ellipse

(c) Hyperbola

(d) Circle

4. A parametrization of the line segment with initial point $(0, 1)$ and terminal point $(1, 0)$ is

(a) $x = \sin^2 t$, $y = \cos^2 t$, $0 \leq t \leq \frac{\pi}{2}$

(b) $x = 1 - t$, $y = t$, $0 \leq t \leq 1$

(c) $x = t$, $y = 1 + t$, $-1 \leq t \leq 0$

(d) $x = 1 + t$, $y = t$, $0 \leq t \leq 1$

5. The Parametrization $x(t) = -\cos t$, $y(t) = \sin t$, $0 \leq t \leq 2\pi$

- (a) A circle with initial point $(1,0)$ counterclockwise.
- (b) A circle with initial point $(-1,0)$ counterclockwise.
- (c) A circle with initial point $(-1,0)$ clockwise.
- (d) A circle with initial point $(1,0)$ clockwise.

6. The length of the curve $x(t) = \sin t$, $y(t) = 1 + \cos t$, $0 \leq t \leq \frac{\pi}{2}$ is:

- (a) π
- (b) 2π
- (c) $\frac{\pi}{4}$
- (d) $\frac{\pi}{2}$

7. The slope of the tangent line to the curve $y(t) = t^2 - \sin t$, $x(t) = \cos t$, at $t = \frac{\pi}{2}$ is:

- (a) $\frac{\pi^2}{2}$
- (b) $\frac{-1}{\pi}$
- (c) -2
- (d) $-\pi$

8. Which one of the following points lies on the curve $r = \cos 2\theta$

- (a) $(0,0)$
- (b) $(1, \frac{\pi}{2})$
- (c) $(\frac{1}{2}, \frac{\pi}{3})$
- (d) all

9. The graph of $r = 2 \csc \theta$ is

- (a) Circle
- (b) Ellipse
- (c) Hyperbola
- (d) Parabola

10. The curves $\theta = \frac{\pi}{2}$ and $r = 0$

- (a) never meets.
- (b) intersect in one point.
- (c) intersect in infinitely many points.
- (d) are identical.

11. $7^{\log_7 5} =$

- (a) $\frac{5}{7}$
- (b) $\frac{7}{5}$
- (c) 5
- (d) 7

12. The directrix of the parabola $y = 2x^2 + 4x + 1$ is

- (a) $y = -\frac{1}{8}$
- (b) $y = \frac{7}{8}$
- (c) $y = -\frac{9}{8}$
- (d) $y = \frac{9}{8}$

13. The quadratic equation $x^2 + 2xy + y^2 - 2x + 2y + 10 = 0$ represents

- (a) Parabola
- (b) Ellipse
- (c) Hyperbola
- (d) Circle

14. A parametrization of the line segment with initial point (0,1) and terminal point (1,0) is

- (a) $x = \sin^2 t$, $y = \cos^2 t$, $0 \leq t \leq \frac{\pi}{2}$
- (b) $x = 1 - t$, $y = t$, $0 \leq t \leq 1$
- (c) $x = 1$, $y = 1 + t$, $-1 \leq t \leq 0$
- (d) $x = 1 + t$, $y = 1$, $0 \leq t \leq 1$

15. $\frac{\log_2(x)}{\log_8(x)} =$

- (a) $\frac{2}{8}$
- (b) $\frac{1}{3}$
- (c) \log_4^x
- (d) 3

16. $\sec^{-1}(-2)$

- (a) $\frac{2\pi}{3}$
- (b) $-\frac{\pi}{3}$
- (c) $-\frac{2\pi}{3}$
- (d) $\frac{3\pi}{4}$

17. $\lim_{x \rightarrow 0^+} (\ln x - \ln(\sin x))$

- (a) 0
- (b) -1
- (c) +1
- (d) Doesn't exist.

18. $\lim_{x \rightarrow -\infty} (\ln x)^{\frac{1}{2}}$

- (a) ∞
- (b) 0
- (c) 1
- (d) Doesn't exist

19. $\lim_{x \rightarrow \frac{\pi}{2}^-} \frac{\sec x}{\tan x} =$

- (a) 0
- (b) 1
- (c) ∞
- (d) Doesn't exist

20. The order of the functions $x^2, e^x, \ln x$ from slowest growing to fastest growing as $x \rightarrow 0$ is

- (a) $x^2, \ln x, e^x$
- (b) $e^x, x^2, \ln x$
- (c) $x^2, e^x, \ln x$
- (d) $\ln x, x^2, e^x$

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II (20 points): Suppose the path of a moving particle in a plane is described by

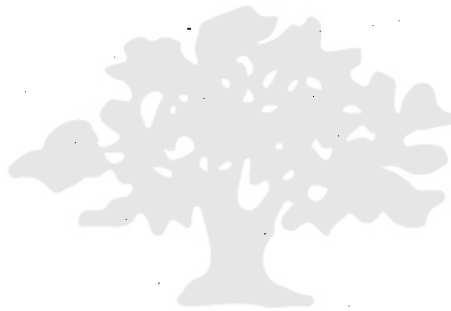
$$\begin{aligned}x(t) &= 3 + 4 \sin(t) \\y(t) &= 2 + 5 \cos(t), \quad 0 \leq t \leq \pi\end{aligned}$$

1. Sketch the path of motion and determine the direction.
2. Find the equation of the tangent line at $t = \frac{\pi}{3}$.

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Question #4:

Graph the conic section $2x^2 + 3xy + 2y^2 - 1 = 0$ in the xy -plane indicating the center, the vertices and the foci in the xy coordinates.

$$2x^2 + 3xy + 2y^2 - 1 = 0$$

$$B^2 - 4AC = 9 - 4(2)(2) < 0 \text{ ellipse}$$

$$\cot 2\alpha = \frac{A-C}{B} = \frac{2-2}{3} = 0$$

$$2\alpha = \frac{\pi}{2}$$

$$\alpha = \frac{\pi}{4}$$

$$x = x' \cos \frac{\pi}{4} - y' \sin \frac{\pi}{4}$$

$$y = \frac{x'}{\sqrt{2}} + \frac{y'}{\sqrt{2}}$$

$$y = x' \sin \frac{\pi}{4} + y' \cos \frac{\pi}{4}$$

$$= \frac{x'}{\sqrt{2}} + \frac{y'}{\sqrt{2}}$$

Equation became

$$2 \left(\frac{x' - y'}{\sqrt{2}} \right)^2 + 3 \left(\frac{x' - y'}{\sqrt{2}} \right) \left(\frac{x' + y'}{\sqrt{2}} \right) + 2 \left(\frac{x' + y'}{\sqrt{2}} \right)^2 - 1 = 0$$

$$\frac{2(x'^2 + y'^2 - 2x'y')}{2} + 3 \left(\frac{x'^2 - y'^2}{2} \right) + 2 \left(\frac{x'^2 + y'^2 + 2x'y'}{2} \right) - 1 = 0$$

$$x'^2 + y'^2 + \frac{3x'^2}{2} - \frac{3y'^2}{2} + x'^2 + y'^2 = 1$$

$$\frac{7}{2}x'^2 + \frac{1}{2}y'^2 = 1$$

$$\frac{x'^2}{\frac{2}{7}} + \frac{y'^2}{2} = 1$$

$$\frac{x'^2}{\frac{2}{7}} + \frac{y'^2}{2} = 1$$

Foci

$$c = \sqrt{a^2 - b^2} = \sqrt{\frac{2}{7} - 2} = \sqrt{-\frac{12}{7}}$$

$$c = \sqrt{a^2 - b^2} = \sqrt{2 - \frac{2}{7}} = \sqrt{1.714} = 1.3$$

Foci at $x'y'$ plan = $(0, \pm 1.3)$

at xy plan

$$y = x' \sin \frac{\pi}{4} + y' \cos \frac{\pi}{4}$$

$$= 0 + 1.3 \cos \frac{\pi}{4}$$

$$= \frac{1.3}{\sqrt{2}}$$

$$x = x' \cos \frac{\pi}{4} - y' \sin \frac{\pi}{4}$$

Foci in xy plan = $\left(\frac{1.3}{\sqrt{2}}, \frac{1.3}{\sqrt{2}} \right)$
 $\left(\frac{1.3}{\sqrt{2}}, -\frac{1.3}{\sqrt{2}} \right)$



~~vertices~~
vertices

$$a = (0, \pm\sqrt{2})$$

$$x = x' \cos \alpha - y' \sin \alpha$$

$$x = 0 - \frac{\sqrt{2}}{\sqrt{2}} = -1$$

3 vertices in R_{xy} plane

~~$a_1 = (1, 1)$~~

$$a_1 = (-1, 1)$$

$$a_2 = (1, -1)$$

$$y = x' \sin \alpha + y' \cos \alpha$$

$$= 0 + \frac{\sqrt{2}}{\sqrt{2}} = 1$$

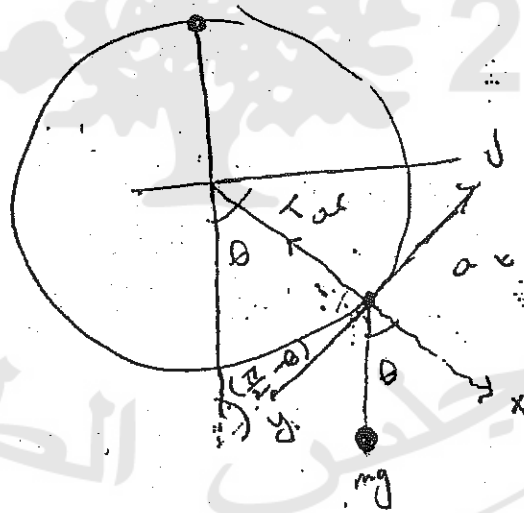
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$$\frac{mv^2}{R} = \frac{T}{\sin \theta}$$

$$v = \sqrt{Rg \sin \theta}$$



$$\frac{mv^2}{R}$$

$$a = -\frac{mg \sin \theta}{T}$$

$$mg \cos \theta + \frac{mv^2}{R} = T \quad (1)$$

s a b'

Q#7:

Find the series radius of convergence . For what values of x does the series converge :

a) Conditionally

b) absolutely

(i) $\frac{(x-2)^n}{n}$

(ii) $\frac{(-1)^n x^n}{n!}$

Q#8:

a) Find the maclurin series for $f(x) = \ln(1+x^2)$.

b) How many terms are supposed to be used to get an estimate $f(0.2)$ with error less that 10^{-6} .

#9:

Evaluate: $\sum_{n=1}^s nx^n$ if $|x| < 1$

MATH DEPARTMENT

MATH 132 TEST THREE

TIME: 60 Min.

JANUARY 2008

NAME: George Hannouch

NUMBER: 106 15 15

SECTION: 1

Instructor's name: Dr. Rimon Jad'ouh

82

QUESTION ONE: (MULTIPLE CHOICE) [40 POINTS]
CIRCLE THE RIGHT ANSWER:

28
30
10
14

1. Let $f(x) = \sum_{n=0}^{\infty} x^n$. The interval of convergence of the definite integral 0 to x,

$\int_0^x f(t) dt$ is

$$\frac{x}{n+1} = \frac{x^{n+1}}{x+1} - \frac{x^n}{x+1}$$

- (A) $x = 0$ only
- (B) $|x| \leq 1$
- (C) $-\infty < x < \infty$
- (D) $-1 \leq x < 1$
- (E) $-1 < x < 1$

$\ln |1+x|$

2. The coefficient of x^4 in the Maclaurin series for $f(x) = e^{-x/2}$ is

- (A) $-1/24$
- (B) $1/24$
- (C) $1/96$
- (D) $-1/384$
- (E) $1/384$

$$\frac{(-1)^n (x)^n}{n!} = \frac{(-1)^4 x^4}{4!} = \frac{1}{24}$$

$$\frac{1}{e^{x/2}} = \sum_{n=0}^{\infty} \frac{(-1)^n (x/2)^n}{n!}$$

3. Which of the following series diverges?

- (A) $\sum 1/n^2$
- (B) $\sum 1/(n^2 + n)$
- (C) $\sum n/(n^3 + 1)$
- (D) $\sum \frac{n}{\sqrt{4n^2 - 1}}$
- (E) none of the preceding.

conv
conv.

$$\frac{n}{\sqrt{4n^2 - 1}} \sim \frac{1}{2}$$

5

4. For which of the following series does the Ratio Test fail?

- (A) $\sum 1/n!$
- (B) $\sum n/2^n$
- (C) $1 + 1/2^{3/2} + 1/3^{3/2} + 1/4^{3/2} + \dots$
- (D) $(\ln 2)/2^2 + (\ln 3)/2^3 + (\ln 4)/2^4 + \dots$
- (E) $\sum n^n/n!$

Handwritten notes for question 4:

- $(\frac{1}{2^{3/2}})^{3/2}$
- $\frac{1}{3^{3/2}} \times 2^{3/2}$
- $\frac{\ln 3}{2^3} \frac{2^3}{\ln 2}$
- $\frac{1}{2} \frac{\ln 4}{\ln 3}$
- $(\frac{2}{3})^{3/2}$
- $(\frac{3}{4})^{3/2}$

5. Which of the following alternating series diverges?

- (A) $\sum (-1)^{n-1}/n$
- (B) $\sum (-1)^{n+1}(n-1)/(n+1)$
- (C) $\sum (-1)^{n+1}/\ln(n+1)$
- (D) $\sum \frac{(-1)^{n-1}}{\sqrt{n}}$
- (E) $\sum (-1)^{n-1}n/n^2 + 1$

Handwritten note for question 5: $2\sqrt{2n}$

6. Which of the following series converges conditionally?

- (A) $3 - 1 + 1/9 - 1/27 + \dots$
- (B) $\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} - \dots$
- (C) $1/2^2 - 1/3^2 + 1/4^2 - \dots$
- (D) $1 - 1.1 + 1.21 - 1.332 + \dots$
- (E) $1/(1*2) - 1/(2*3) + 1/(3*4) - 1/(4*5) + \dots$

Handwritten notes for question 6: $\frac{1}{3}, \frac{1}{9}, \frac{1}{3}$

7. Suppose $f(x)$ is a function with Taylor series converging to $f(x)$ for all $x \in \mathbb{R}$.

If $f(0) = 2$, $f'(0) = 2$ and $f''(0) = 3$ for $n \geq 2$ then $f(x) =$

- (A) $3e^x + 2x - 1$
- (B) $e^{3x} + 2x + 1$
- (C) $e^{3x} - x + 1$
- (D) $3e^x - x - 1$
- (E) $3e^x + 5x + 5$

$$\frac{\ln\left(\frac{1}{n}\right)}{\frac{1}{n}}$$

$$\frac{\frac{1}{n}}{\frac{1}{n^2}}$$

$$\frac{1}{n^3}$$

$$\frac{1}{n^3} \rightarrow \infty$$

8. Which of the following series converge?

- (I) $\sum_{n=1}^{\infty} \frac{\ln(n^{-3})}{n^{-3}}$ (II) $\sum_{n=1}^{\infty} \frac{\ln 3}{3n}$ (III) $\sum_{n=1}^{\infty} \frac{n}{3^n}$

- (A) I only (B) II only (C) III only
 (D) None (E) I and III

9. What is the Taylor series for $f(x) = e^x$ about $x = 1$?

- (A) $\sum_{n=0}^{\infty} \frac{-(x-1)^n}{n!}$ (B) $\sum_{n=0}^{\infty} \frac{-e(x-1)^n}{n!}$ (C) $\sum_{n=0}^{\infty} \frac{(x-1)^n}{n!e}$
 (D) $\sum_{n=0}^{\infty} \frac{e(x-1)^n}{n!}$ (E) $\sum_{n=0}^{\infty} \frac{(x-1)^n}{n!}$

10. Let $\{a_n\}$ be a sequence of positive real numbers such that

$$\frac{1}{2} \leq \frac{a_{n+1}}{a_n} \leq \frac{n+4}{2n+1} \text{ for all } n. \text{ Then } \lim_{n \rightarrow \infty} a_n =$$

- (A) 0 (B) 1/2 (C) 1
 (D) 2 (E) 4

$$\lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n} \leq \frac{1}{2}$$

$\Rightarrow \sum a_n$ converges

$$\Rightarrow \lim_{n \rightarrow \infty} a_n = 0$$

QUESTION TWO: [30 points]

Test the following series for convergence or divergence. State the test you are using, and verify that it applies. Determine whether the convergence is absolute or conditional.

(a) $\sum_{n=3}^{\infty} \frac{1}{n(\ln n)^{100}} \Rightarrow$ Converges by integral test

$$\int_3^{\infty} \frac{1}{n(\ln n)^{100}} = \frac{(\ln n)^{-99}}{-99} \Big|_3^{\infty}$$

$$= \frac{1}{99(\ln 3)^{99}} - \frac{1}{99(\ln 3)^{99}} = \frac{1}{99(\ln 3)^{99}} > 0$$

Integral Converges

(b) $\sum_{n=0}^{\infty} \frac{n^{2n}}{(1+2n^2)^n} \Rightarrow$ Converges by the n th Root test absoli

$$\lim_{n \rightarrow \infty} \sqrt[n]{\frac{n^{2n}}{(1+2n^2)^n}}$$

$$= \lim_{n \rightarrow \infty} \frac{n^2}{1+2n^2} = \boxed{\frac{1}{2}} < 1 \Rightarrow \text{converges}$$

(c) $\sum_{n=1}^{\infty} \frac{n^3 - \sqrt{n} + 6}{n^4 - n - 3}$ diverges by limit comparison test

$$\lim_{n \rightarrow \infty} \frac{n^3 - \sqrt{n} + 6}{n^4 - n - 3} \sim \frac{1}{n}$$

$$\lim_{n \rightarrow \infty} \frac{n^4 - n^{\frac{3}{2}} + 6n}{n^4 - n - 3}$$

$= 1 \Rightarrow$ ~~both~~ both diverge or converg

$\frac{1}{n}$ diverges. (power series with $p=1$)

\Rightarrow both diverg

QUESTION THREE: [14 points]

Consider the power series $\sum_{n=1}^{\infty} \frac{(-2)^n}{\sqrt{n}} (x+3)^n$

- (a) When $x = -4$ does this series converge or diverge?
 (b) Determine all values for which the series converges.

$$\lim_{n \rightarrow \infty} \sqrt[n]{\frac{(-2)^n}{\sqrt{n}} (x+3)^n}$$

$$= \frac{-2}{\sqrt[n]{n}} (x+3) \quad \therefore |x+3| \leq \frac{1}{2} \Rightarrow R = \frac{1}{2}$$

$$-1 < x+3 < 1$$

$$\underline{-4} < x < \underline{-2}$$

when $x = -4$

$$\sum \frac{(-2)^n (-1)^n}{\sqrt{n}}$$

$$\sum \frac{2^n}{\sqrt{n}}$$

$$\lim_{n \rightarrow \infty} \frac{2^n}{\sqrt{n}} \div \frac{1}{\sqrt{n}}$$

$$\lim_{n \rightarrow \infty} 2^n = \infty$$

both diverg
 by L.C.T

when $x = -2$

$$\sum \frac{(-2)^n}{\sqrt{n}}$$

Converges conditionally b A.S.T

\Rightarrow the series converges on the interval $(-4, -2]$

QUESTION FOUR: [16 points]

Consider the integral $\int x \cos(x^3) dx$.

- (a) Write down the Maclaurin series for $\cos(x)$, $\cos(x^3)$, and $x \cos(x^3)$.
 (b) Evaluate $\int x \cos(x^3) dx$ as an infinite series.

$$\cos(x) = \frac{(-1)^n x^{2n}}{2n!}$$

$$\cos(x^3) = \frac{(-1)^n x^{6n}}{2n!}$$

Maclaurin

$$\cos(x^3) = 1 + \frac{x^6}{2!} + \frac{x^{12}}{4!} + \dots$$

$$\cos(x^3) = 1 + \frac{x^6}{2!} + \frac{x^{12}}{4!} + \frac{x^{18}}{6!} + \dots$$

$$x \cos(x^3) = x + \frac{x^7}{2!} + \frac{x^{13}}{4!} + \frac{x^{19}}{6!} + \dots$$

$$\int_0^1 x \cos(x^3) dx = x + \frac{x^7}{2!} + \frac{x^{13}}{4!} + \frac{x^{19}}{6!} + \frac{x^{25}}{8!} + \dots$$

$$\int_0^1 x \cos(x^3) dx = \frac{2x + x^7}{2} + \dots$$

$$\int_0^1 x \cos(x^3) dx = \frac{2}{3} + \dots$$

Birzeit University
Department of Mathematics
Math 132

Key

Final exam

Name :

Instructor:

Summer/2009

Number: ...

Section :

Q#1 (72%) Circle the correct answer.

$$\ln \frac{H}{V} x = \frac{b}{\pi}$$

1) $\int_{\frac{1}{\sqrt{3}}}^{\sqrt{3}} \frac{dx}{(1+x^2) \tan^{-1} x} =$

a) $\ln 4 - \ln 3$

c) $\ln 3 - \ln 2$

b) $\ln 2$

d) $\frac{\pi}{12}$

2) If $y = (\ln x)^x$ then $\frac{dy}{dx} =$

a) $(\ln x)^x \left(\frac{1}{\ln x} + \ln x \right)$

c) $x (\ln x)^{x-1}$

b) $(\ln x)^{x-1}$

d) $(\ln x)^x \left[\frac{1}{\ln x} + \ln(\ln x) \right]$

3) $\int_1^{e^3} \frac{1}{x \sqrt{1 + \ln x}} dx =$

a) $\ln \frac{4}{3}$

b) 2

c) $\frac{4}{3}$

d) $\ln \frac{3}{2}$

4) $\int_1^2 \frac{\sinh(\ln x)}{x} dx =$

a) 0

c) $\frac{1}{4}$

b) 1

d) $\frac{1}{2}$

5) if $y = \tan^{-1}\left(\frac{1}{x}\right)$ then $\csc y =$

a) $\frac{\sqrt{1+x^2}}{x}$

b) $\sqrt{1+x^2}$

c) $\frac{x}{\sqrt{1+x^2}}$

d) $\frac{1}{\sqrt{1+x^2}}$

6) if $y = 5^{\ln x}$ then $\frac{dy}{dx}$ when $x=1$ is:

a) 0

b) $-\ln 5$

c) $\ln 5$

d) 1

7) $\int_0^1 e^{\sqrt{x}} dx =$

a) 0

b) 4

c) 3

d) 2

8) $\int x^2 e^{3x} dx =$

a) $\frac{1}{3}x^2 e^{3x} + \frac{2}{9}x e^{3x} + \frac{2}{27}e^{3x} + c$

b) $\frac{1}{3}x^2 e^{3x} + \frac{2}{9}x e^{3x} - \frac{2}{27}e^{3x} + c$

c) $\frac{1}{3}x^2 e^{3x} - \frac{2}{9}x e^{3x} + \frac{2}{27}e^{3x} + c$

d) $-\frac{1}{3}x^2 e^{3x} + \frac{2}{9}x e^{3x} - \frac{2}{27}e^{3x} + c$

9) $\int \frac{dx}{(x+2)\sqrt{x^2+4x}} =$

a) $\sqrt{x^2+4x} + c$

b) $\frac{1}{2} \sec^{-1} \left| \frac{x+2}{2} \right| + c$

c) $\frac{1}{2} \ln |x^2+2x| + c$

d) $\sinh^{-1}(x+2) + c$

10) If $f(x) = xe^x$, then $(f^{-1})'(e) =$

- (a) $\frac{1}{e^e(e+1)}$ (b) $e^e + ee^e$ (c) $\frac{1}{2e}$ (d) None of the above

$x e^x \rightarrow e^x$

11) Consider the improper integrals:

(i) $\int_3^{\infty} \frac{dx}{(x-3)^2}$ (ii) $\int_{-5}^{\infty} \frac{dx}{\sqrt{x+5}}$

- (a) only integral (i) converge (b) only integral (ii) converges
 (b) both integrals converge (d) both integrals diverge.

12. The power series $x + \frac{x^2}{2} + \frac{x^3}{3} + \dots + \frac{x^n}{n} + \dots$ converges if and only if

- (a) $-1 < x < 1$ (b) $-1 \leq x \leq 1$
 (c) $-1 \leq x < 1$ (d) $-1 < x \leq 1$

$\frac{x}{1-x} = 1 + x + x^2 + \dots + x^{n-1}$

$(1+x)^{\frac{1}{2}} = 1 + \frac{1}{2}x - \frac{1}{8}x^2$

13. The Maclaurin series of order 3 for $f(x) = \sqrt{x+1}$ is

- (a) $1 + \frac{x}{2} - \frac{x^2}{4} + \frac{3x^3}{8}$ (b) $1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16}$
 (c) $1 - \frac{x}{2} + \frac{x^2}{8} - \frac{x^3}{16}$ (d) $1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{8}$

??

14) Determine whether $\int_2^{30} \frac{dx}{(x-3)^{2/3}}$ converges or diverges. If the integral

- converges, find its value
 a) converges, 15
 c) converges, 1

- b) converges, 3
 d) diverges.

$\int_2^{30} \frac{dx}{(x-3)^{2/3}} = \int_2^{30} (x-3)^{-2/3} dx$
 $= 3 \sqrt[3]{x-3} \Big|_2^{30}$
 $= 3 \sqrt[3]{27} - 3 \sqrt[3]{-1}$
 $= 9 - (-3) = 12$

15) If $(\frac{1+i}{1-i})^4 + z = 2+i$ then $z =$

a) $2-i$

b) $1-2i$

c) $2+4i$

d) $1+i$

16) The series $\sum_1^{\infty} \frac{n^n}{2^n 3^n}$

a) Diverges by Ratio test

b) Converges by n^{th} term test

c) Converges by Integral test

d) Converges by n^{th} root test

17) The series $\sum_1^{\infty} (-1)^n (\frac{n^2}{2n^4 + 5})$

a) Converges by limit comparison test

b) Diverges by n^{th} term test

c) Converges by n^{th} term test

d) Converges absolutely

18) The sequence $\{a_n\} = \{ \frac{1+(-1)^n}{n} \}$

a) Converges to 1

b) Converges to 0

c) Converges to 2

d) Diverges

19) $\int \sin^{-1} x \, dx =$

a) $x \sin^{-1} x - 2\sqrt{1-x^2} + c$

b) $x \sin^{-1} x + 2\sqrt{1-x^2} + c$

c) $x \sin^{-1} x - \sqrt{1-x^2} + c$

d) $x \sin^{-1} x + \sqrt{1-x^2} + c$

$u = \sin^{-1} x \quad dv = dx$
 $du = \frac{1}{\sqrt{1-x^2}} dx \quad v = x$

$x \sin^{-1} x + \frac{1}{2} \int \frac{-2x \, dx}{\sqrt{1-x^2}}$

$$20) \int \frac{dx}{\sqrt{2x-x^2}} =$$

a) $2\sqrt{2x-x^2} + c$

c) $\sin^{-1}(x-2) + c$

b) $\sin^{-1}(x-1) + c$

d) $\sec^{-1}(x-1) + c$

21) The series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt{(n+1)(n+2)}}$

a) Converge conditionally

c) Diverges

b) Converge absolutely

d) Converges to 2

22) $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$

a) Converges Absolutely

c) Diverges by alternating series theorem

b) Converges conditionally

d) Diverges by n^{th} -term test

$e^x = e^x$

23) If $\cosh x + \sinh x = e$ then $x =$

a) e

c) $\ln 5$

b) 1

d) $\ln \sqrt{5}$

24) $\int_3^4 \frac{3dx}{x^2+x-2} =$

a) $\ln \frac{5}{4}$

b) $\ln \frac{4}{5}$

c) $\ln \frac{8}{5}$

d) $\ln \frac{5}{8}$

$\int \frac{1}{x-1} - \frac{1}{x+2}$

$\ln \frac{x-1}{x+2} \Big|_3^4 = \ln \frac{3}{5} \times \frac{5}{2}$

Q2(9%) Use series to find an estimate for $\int_0^{\frac{1}{2}} \frac{\tan^{-1}x}{x} dx$ with an error of magnitude less than 10^{-3}



Q3(10%) Solve $z^4 = -81$ in the field of complex numbers



Q4)(10%) Sketch the graph of the conic section

$$9x^2 + 25y^2 + 18x - 100y = 116 \quad \text{indicating}$$

a) foci

b) vertices

c) eccentricity.....

d) directrices.....

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